A STUDY OF REACTIVITY CHANGES IN THE AGN-201 REACTOR USING PERTURBATION THEORY

by

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THESIS

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ABSTRACT

The effects of fuel burnup, fission product poisoning, and hydrogen moderator density variation on reactivity in the AGN-201 reactor are considered. A modified one-group perturbation theory is developed and applied to changes in parameters resulting from a change in hydrogen moderator density. An equation for the reactivity change is obtained for three models: a bare cylindrical core, a bare core using extrapolated dimensions, and a reflected core. These three equations are then used to predict values of the reactivity increase resulting from interchanging a new 3/4 inch thick fuel disk with comparable fuel disks presently in the core.

The results obtained by a digital computer solution of the reactivity equations reveal that the increase in reactivity varies from 0.4392 to 0.7707%, depending upon the core model and position of the old disk within the core. Because the license of the Naval Postgraduate School does not permit a value of excess reactivity above 0.40%, it is concluded that a simple interchange of disks in this manner would produce too large a value of excess reactivity.

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NOMENCLATURE

ENGLISH SYMBOLS

A^p = molecular weight of polyethylene

A^u = atomic weight of uranium

B = buckling of the reactor system

d = extrapolation distance

D = diffusion coefficient of core

D_r = diffusion coefficient of reflector

f = thermal utilization

g = non-1/v factor

h = distance of nearest edge of fuel disk above or below the zero ordinate of the z-axis of the cylindrical fuel core

H = height of fuel core

H^0 = extrapolated height of fuel core

I = effective resonance integral

K_∞ = multiplication factor for an infinite reactor system

L = diffusion length in core

L_r = diffusion length in reflector

M = perturbation operator

M+ = adjoint of M

M^0 = perturbation operator of perturbed reactor system

M_f = mass of fuel per disk

M_p = mass of polyethylene

N = atom density

N_a = Avogadro's number

N_o = atom density at time equal to zero

P = resonance escape probability

Pm = promethium concentration

PWR = reactor power

R = radius of fuel core

R' = extrapolated radius of fuel core

S = samarium concentration

S_∞ = equilibrium samarium concentration

t = time

T = reflector thickness

T_{eff} = effective neutron temperature

T_m = moderator temperature

T_r = radial reflector thickness

T_z = end reflector thickness

V_d = disk volume

Z_d = centerline distance from a disk to the zero ordinate of the z-axis of the fuel core

GREEK SYMBOLS

γ_p = fission-product poison yield of promethium

δ = reflector savings

δh = disk thickness

δr = radial reflector savings

δz = end reflector savings

Δ_n = fraction of total change in core moderator mass occurring in a fuel disk

ϵ = fast fission factor

η = net neutrons available after fission

λ_p = decay constant of promethium

λ_{tr} = transport mean free path

ν = average number of neutrons released per fission

ξ = average logarithmic energy decrement per collision

ρ = reactivity

ρ^{p} = density of polyethylene

ρ^{u} = density of uranium

ρ^{235} = density of U-235

ρ^{238} = density of U-238

σ_a = microscopic absorption cross-section

σ_f = microscopic fission cross-section

σ_s = microscopic scattering cross-section

Σ_a = macroscopic absorption cross-section

Σ_f = macroscopic fission cross-section

Σ_s = macroscopic scattering cross-section

τ_o = Fermi Age

ϕ = neutron flux

$\bar{\phi}$ = average neutron flux

ϕ_o = peak neutron flux

ϕ' = neutron flux of perturbed reactor system

ψ = importance function

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I. INTRODUCTION

The AGN-201 nuclear reactor is a small, safe, low-power, self-contained reactor designed and produced by Aerojet-General Nucleonics Corporation of San Ramon, California. It is designed primarily for use in education and research not requiring a high neutron flux, but featuring maximum safety, high analytical sensitivity, and ready accessibility of components. The fuel core of the reactor is composed of ten circular fuel disks of varying thicknesses, piled on top of one another to form a cylindrical shaped core. The fuel disks are made of polyethylene and have uranium fuel homogeneously interspersed in them. Appendix A contains a general physical description of the overall reactor and also a detailed discussion of its fuel core.

The AGN-201 nuclear reactor located at the Naval Postgraduate School, Monterey, California, has been in licensed operation since April 29, 1957. From that date, until January 31, 1962, the reactor was operated at a maximum power level of 100 milliwatts. During that time period no decrease in excess reactivity was noted. Thereafter, the license of the Naval Postgraduate School was amended to permit continuous operation up to 20 watts and intermittent operation at power levels up to 1000 watts. On February 3, 1962, the excess reactivity of the reactor was measured and recorded as 0.354%. From that time until February 26, 1969, the value of the excess reactivity has steadily decreased to a value of 0.281% or a drop of 0.073%.

Several reasons which would account for the observed loss in reactivity have been postulated. The first cause of this loss could be due to fuel burnup. Since the reactor is highly enriched with uranium-235,

(approximately 20%), the decrease in reactivity would be proportional to the fraction of U-235 depleted by fission. A second cause of reactivity drop could be the buildup of reactor poisons due to fission of the fuel. These fission fragments and their decay products have large neutron absorption cross-sections. If they are produced in appreciable amounts they could effectively alter the neutron balance in the reactor by affecting neutron multiplication. Thirdly, the loss in reactivity of the reactor might be attributed to a loss in moderator within the reactor core. This problem is unique to this type of reactor. Polyethylene, which is the neutron moderator, when subjected to radiation, tends to decompose. This decomposition is accompanied by the liberation of hydrogen gas. The effect of this phenomenon on the reactivity is especially pronounced because the hydrogen is bled off from the fuel core at varying time intervals which reduces the density of the moderator within the core.

In September, 1968, the Naval Postgraduate School received a new fuel core disk for the purpose of adding it to the core in some manner to attempt to raise the value of the excess reactivity of the reactor. However, the limitation imposed upon this addition is that the resulting value of the excess reactivity of the reactor must not exceed 0.40%. The new disk has a known fuel enrichment and a higher polyethylene density than any of the disks presently in the core. The properties of the new disk are presented in Appendix A.

The objective of this thesis is to determine the change in reactivity which would result from interchanging the new fuel disk with a disk of comparable size currently in the reactor. To accomplish this objective, first an analysis is made of the three possible causes of the reactivity loss due to reactor operation to determine which have significant effects,

and what changes in reactor properties result from these effects. Second, using a modified one-group perturbation theory, an equation for the reactivity as a function of these changes is derived for three models describing the reactor core. These models treat the core separately as a bare cylinder, a bare cylinder having extrapolated dimensions, and a reflected cylinder. Third, a digital computer program is used to solve the reactivity equations using assumed changes in reactor properties. Since the actual value of reactivity drop due to operation is known, the core properties at present can be found. Using these results and the properties of the new fuel disk, the reactivity equations derived from perturbation theory are solved by a second computer program to yield values of the excess reactivity caused by the perturbation of replacing an old disk with the new disk.

II. VARIABLES WHICH AFFECT REACTIVITY

A. FUEL BURNUP

The first of the three probable causes of decrease in excess reactivity to be considered is that of fuel burnup. From published literature concerning the AGN-201 reactor [1][1], it is predicted that continuous reactor operation (24 hours a day, 7 days per week) at a 100 milliwatt power level would result only in a burnup rate of approximately 30 micrograms/year. On the basis of this information it can probably be anticipated that even at higher power levels, the contribution of fuel burnup to a reactivity drop will be slight, if any.

Since the reactor is fueled with only a single fissile substance, U-235, and there are no fertile materials present, (it is assumed that due to the high enrichment of U-235 there is no compensation for fuel depletion through the production of fissile Pu-239 from neutron absorption by U-238), the depletion rate of the fuel is governed by the equation

$$\frac{dN}{dt} = -N\bar{\sigma}_a\phi \qquad (1)$$

where N is the atom density of the fuel, $\bar{\sigma}_a$ is the thermal absorption cross-section, and ϕ is the neutron flux. Rearranging and integrating (1) yields

$$\frac{N}{N_o} = e^{-\int_o^t \bar{\sigma}_a \phi t\, dt}$$

The integral exponent, $\int_o^t \bar{\sigma}_a \phi t\, dt$, can be expressed as $\bar{\sigma}_a \bar{\phi}\, t$, where $\bar{\phi}$ is an average value of the flux over time t. Then

[1]Numbers in brackets refer to Bibliography.

$$\frac{N}{N_o} = e^{-\bar{\sigma}_a \bar{\phi} t} \qquad (2)$$

From recorded data contained in the operator's log kept at the reactor facility, as of December 9, 1968, the reactor had been operated approximately 8.69×10^5 watt-min. Although operation occurred at power levels varying from 0.1 watt to 1000 watts, 20 watts was most frequent, and therefore was assumed to be a representative average value. This gives an average time of operation of

$$t = 2.61 \times 10^6 \text{ sec.}$$

An equation given by Lamarsh [5] relating reactor power and neutron flux, assuming a recoverable energy of 200 Mev per U-235 fission is

$$\bar{\phi} = \frac{(PWR)(T_{eff})^{1/2} \times 10^{13}}{(7.19)(M_f)(g_f)} \qquad (3)$$

where PWR is the reactor power, T_{eff} is the effective neutron temperature, M_f is the mass of fuel, and g_f is the non-$1/v$ factor. Since

$$PWR = 20 \times 10^{-6} \text{ megawatts}$$

$$T_{eff} = 340^\circ \text{ K}$$

$$g_f(T) = 0.9269$$

and $$M_f = 0.667 \text{ Kg}$$

evaluation of equation (3) yields

$$\bar{\phi} = 8.3 \times 10^8 \text{ neutrons/cm}^2\text{-sec}$$

and the quantity

$$\bar{\sigma}_a \bar{\phi} t = 1.16 \times 10^{-6} \qquad (4)$$

where

$$\bar{\sigma}_a = \frac{1}{1.128} \sqrt{\frac{T_m}{T_{eff}}}\, \sigma_a\, g_a(T)$$

Substituting (4) into (2),

$$\frac{N}{N_o} = e^{-(1.16 \times 10^{-6})} = 0.9999988$$

or $$\frac{N}{N_o} \approx 1.$$

Therefore it is concluded that fuel burnup is negligible at the assumed average power level of 20 watts.

B. FISSION PRODUCT POISONING

As previously mentioned, reactor poisons, having large neutron absorption cross-sections, have an adverse effect on reactivity. Their chief influence on reactivity is through the multiplication factor by decreasing the thermal utilization. In discussing reactor poisons in the AGN-201, it is felt that in order to affect reactivity, a poison produced in this particular reactor must meet three criteria. (1) It must have a high cross-section. (2) It must have a solid form. (3) It must be stable. Although as many as 200 species are apt to be produced during the fission process, only two have large enough cross-sections and are produced in sufficient quantity to warrant attention; namely Xe-135 and Sm-149.

In analyzing these two, it is seen that Xe-135 meets neither condition (2) nor (3) above. Being an inert gas, it would be removed from the reactor at the time when the hydrogen buildup is bled off from the core. Thus the decrease in reactivity would be a function of time, being zero at a time equal to zero, buildup to some value, and then return to zero at the time of the hydrogen and poison removal. However, this is contrary to observed evidence of reactivity behavior. In addition, Xe-135 is not stable but is an intermediate isotope in the chain

$$Te^{135} \longrightarrow I^{135} \longrightarrow Xe^{135} \longrightarrow Cs^{135} \longrightarrow Ba^{135} \quad \text{(stable)}.$$

Because it is not a stable isotope, any xenon poison produced during operation would eventually decay after reactor shutdown, resulting in

no long term effect on the reactivity. Hence, Xe-135 can be effectively ruled out as a cause of long term reactivity drop.

Sm-149, on the other hand, meets all three requirements. It has a thermal absorption cross-section equal to 58,500 barns at 20°C. It is a rare earth metal and is the stable decay product of the chain

$$Nd^{149} \longrightarrow Pm^{149} \longrightarrow Sm^{149} \quad \text{(stable)}.$$

The neodynium decays relatively quickly compared to the promethium so that for practical purposes the Pm-149 may be assumed to be produced directly as a result of fission. The concentration of Pm-149 is then governed by the equation

$$\frac{dPm}{dt} = \gamma_p \Sigma_f \phi - \lambda_p Pm \tag{5}$$

where Pm is the promethium concentration, γ_p is the fission-product poison yield for promethium, Σ_f is the fission cross-section of the fuel, ϕ is the neutron flux, and λ_p is the decay constant for promethium.

The rate of change of the concentration of Sm-149 can be expressed as

$$\frac{dS}{dt} = \lambda_p Pm - \sigma_a^S \phi S \tag{6}$$

since samarium is stable and can only be lost through neutron absorption. In equation (6) S is the samarium concentration and σ_a^S is the absorption cross-section of samarium. Combining (5) and (6) and integrating yields an expression for Sm-149 concentration as a function of time.

$$S = \frac{\gamma_p \Sigma_f}{\sigma_a^S} \left(1 - e^{-\lambda_p t}\right)\left(1 - e^{-\sigma_a^S \phi t}\right) \tag{7}$$

Since $\gamma_p = 0.0113$

$\Sigma_f = 0.0646\ cm^{-1}$

$\sigma_a^s = 5.85 \times 10^{-20}\ cm^2$

$\lambda_p = 3.56 \times 10^{-6} sec^{-1}$

and from the previous section it was assumed that

$t = 2.61 \times 10^6$ sec

$\phi = 8.3 \times 10^8$ neutrons/cm^2-sec,

Then

$$S = 1.313 \times 10^{12}\ cm^{-3}.$$

It can be seen from equation (7) that as $t \longrightarrow \infty$, S would no longer be a function of time but would approach a constant "equilibrium concentration". For purposes of comparison let $t \longrightarrow \infty$ in (7). Therefore,

$$S_\infty = \frac{\gamma_p \Sigma_f}{\sigma_a^s} = 1.178 \times 10^{16}\ cm^{-3}.$$

Thus it is seen that the concentration at present is less than the equilibrium value by a factor of approximately 10^4. The macroscopic cross-section of the samarium is

$$\Sigma_a^s = S\sigma_a^s = 7.68 \times 10^{-8}\ cm^{-1}.$$

The change in reactivity due to samarium poisoning can be determined from the equation [5]

$$\delta = \frac{\Sigma_a^s / \Sigma_f}{\nu \epsilon p}$$

where

$\nu = 2.43$

and $p \approx \epsilon \approx 1.$

Thus,

$$\delta = -0.48 \times 10^{-4}\%.$$

Using the same equations with the equilibrium concentration of Sm-149

yields

$$\rho = -0.439\%$$

As can be seen, there is a difference of a factor of 10^4 between the reactivity values. Further, using the same equations to solve for the time required to reach equilibrium concentration yields a value of 8.11 x 10^7 sec. This is a factor of 40 times larger than the time of reactor operation to present.

From this analysis it can be concluded that samarium poisoning does exist in the reactor, but it has not reached its equilibrium value. Due to the relatively short period of operating time and the low flux level produced during operation of the reactor, the decrease in reactivity due to samarium is several orders of magnitude smaller than the observed value of 0.073%. Consequently the effect of samarium will be neglected.

C. MODERATOR CONCENTRATION

The third possible cause of reactivity drop is that of loss of moderator. As already noted, the moderating material in the AGN-201 reactor is polyethylene. Chemically, the molecular formula of polyethylene is $(CH_2)_n$ indicating that it is composed of long chain (high polymer) molecules formed by many (CH_2) groups.

$$-CH_2CH_2CH_2CH_2-$$

In fact, the molecular weight of commercial polyethylene is between 18,000 and 22,000. From a nuclear standpoint, polyethylene is a long series of carbon atoms connected in chains with bare protons (hydrogen nuclei) attached to them. When high energy neutrons are introduced into a polyethylene medium, it is possible for a neutron to collide with the proton and break the proton away from the main chain. When this happens, cross-linking occurs, whereby new bonds are formed between adjacent high

polymer molecules accompanied by the evolution of a hydrogen molecule.

$$\begin{matrix} -CH_2CH_2CH_2CH_2- \\ -CH_2CH_2CH_2CH_2- \end{matrix} \xrightarrow{\sim\sim} \begin{matrix} -CH_2CH_2CH\ CH_2- \\ -CH_2CH_2CH\ CH_2- \end{matrix} + H_2$$

This is referred to as unsaturation of the polyethylene. The main point to be noted is that when polyethylene is irradiated, gas is evolved, the majority of which is hydrogen, along with slight traces of methanes. The assumption made, upon all that follows in this thesis is that due to hydrogen evolution and subsequent removal of the hydrogen from the fuel core, the mass of the polyethylene moderator is decreased and that this decrease is equal in magnitude to the mass of hydrogen removed.

From research conducted with the AGN-201 in March ,1962, shortly after the reactor license was amended to permit higher power operation, it was noted that gas evolution from the core occurs primarily at power levels between 200 and 1000 watts. [7] This gas evolution takes place slowly and over a period of about two days following operation. No gas evolution was observed in relatively short periods of operation at 20 watts. A sampling of the gas revealed traces of an inert gas mixed with the hydrogen, the decay rate of which corresponded to xenon. The fact that the beginning of the observed decrease in excess reactivity coincided with the start of the reactor's operating at high power levels tends to confirm that hydrogen evolution is the cause of the reactivity drop.

In the theory that follows, the decrease in density of the polyethylene moderator manifests itself in two ways. First, it appears directly in the moderator density term, ρ^{p}. Secondly, it causes a change in the fission and absorption cross-sections, σ_a and σ_f, of materials in the reactor. Because the amount of moderator in the fuel core is decreased, the neutrons are not as well moderated and consequently the

average energy of the neutrons is at a higher energy value. When this occurs, the cross-sections of the materials in the reactor, which are energy dependent, are no longer those corresponding to 0.025 ev, but rather those which correspond to a slightly higher energy.

The energy at the peak of the energy curve is related to a temperature known as the effective neutron temperature. The neutron temperature is related to the density of the polyethylene in the following manner. Coveyou [9] has established the relation

$$T_{eff} = T_m (1 + 0.46 \Delta)$$

where

$$\Delta = \frac{4}{\bar{\xi} \Sigma_s} (\Sigma_a^u + \Sigma_a^p)$$

and

$$\Sigma_a^p = \frac{\rho^p N a}{A}$$

for moderators with mass from 1 to 25 and Δ from 0 to 1.0. For heavy moderators, Cohen [9] gives

$$T_{eff} = T_m (1 + 0.3 \Delta).$$

From a survey of research done in the field of polyethylene, Cooke [3] concluded that the effective neutron temperature, T_{eff}, of neutrons in polyethylene was between 336 and 347° K. Since it is known for the reactor that $\Delta = 0.2931$ and $T_m = 293°$ K, and T_{eff} was assumed to be 340° K, the numerical coefficient in the expression for T_{eff} was modified to a value of 0.5459 to fit the polyethylene data. The resulting equation is thus,

$$T_{eff} = T_m (1 + 0.5459 \Delta). \qquad (8)$$

From the equation linking the density of polyethylene to the effective neutron temperature, the neutron temperature can in turn be used to find the cross-section at that temperature through the following equation,

$$\bar{\sigma} = \frac{1}{1.128}\sqrt{\frac{T_m}{T_{eff}}}\ \sigma\, g(T)$$

Thus, from these equations it can be seen that the effect of decreasing the polyethylene density is to increase the neutron temperature which decreases the neutron cross-section of materials in the reactor.

From work done with the reactor in June, 1963, based on test runs of 1000 watt-min, the rate of hydrogen evolution from the fuel core was found to be about 0.72 moles/KW-hr. [8] The number of kilowatt-hours of operation from February, 1962, to February, 1969, the period during which the drop in excess reactivity has occurred, is known to be about 16.07 KW-hr. Therefore, the number of moles, and consequently the weight of hydrogen evolved can be approximated. This results in a value of hydrogen weight of 23.14 gm which when equated to a change in polyethylene mass in the core gives a value which can be used to check against that predicted by perturbation theory.

D. CONCLUDING STATEMENTS

The conclusions drawn from the preceding sections can be summarized as follows. Fuel burnup can be neglected as a cause of reactivity decrease on the grounds that the time of operation of the reactor has not been long enough nor is the assumed average level of operation at 20 watts high enough to cause any significant depletion of the U-235 fuel in the reactor core. The two reactor poisons most likely to be produced during operation of the AGN-201, xenon and samarium, cannot account for the drop in excess reactivity. The Xe-135 is either withdrawn from the reactor simultaneously with the hydrogen during bleedoff or decays to a more stable isotope during the frequent periods of reactor shutdown. The

extremely small concentration of Sm-149 present in the reactor does produce a decrease in the excess reactivity but its magnitude is several orders smaller than the observed decrease and is negligible. The decrease in polyethylene moderator due to hydrogen evolution resulting from neutron irradiation cannot be ruled out as a cause of reactivity drop. It is assumed that this cause is solely responsible for the decrease in reactivity. Also, observed evidence seems to indicate that this is the most probable cause of the reactivity decrease. The change in polyethylene density will appear in the theoretical equations that follow both as a direct density change and also as changes in neutron cross-sections of reactor materials. This is because the cross-sections are functions of neutron temperature which in turn is a function of moderator density.

III. PERTURBATION THEORY

A. INTRODUCTION

To analyze the effect on the behavior of a nuclear reactor caused by small non-uniform changes such as those discussed in the preceding paragraphs, a method known as perturbation theory has been developed. If the problem, which has been outlined, involving the AGN-201 were of such a nature that uniform changes occurred throughout the reactor, then the reactivity could simply be found by recomputing new reactor constants and recalculating a new multiplication factor to find the resulting reactivity. When non-uniform changes within the reactor occur, as is usually the case in practice, then perturbation theory must be employed. Examples of non-uniform perturbations include insertion or withdrawl of control rods into the reactor, non-uniform fuel burnup, non-uniform fission poison accumulation or as it is believed in this case, non-uniform removal of moderator from the core. The assumption upon which perturbation theory is based is that the perturbations or changes within the reactor are small and that the flux is not distorted substantially in the region of the perturbation. Since this is assumed to be the case in analyzing the AGN-201, perturbation theory provides a convenient method of handling an otherwise difficult problem.

B. MODIFIED CRITICALITY EQUATION

Perturbation theory is referred to as one-group, two-group, or many-group depending upon the type of theory used to describe the energy of the neutrons in a critical reactor. The simplest is one-group and will be used here. The criticality equation describing a one-group reactor is

$$\frac{K_\infty}{1 + L^2 B^2} = 1$$

where $K_\infty = \eta f$

Strictly speaking, this equation is used in a one-group perturbation analysis. However, it was felt that improved accuracy of the results could be obtained by using the Fermi Age-Diffusion model to include neutrons in the slowing down region. The usual form of the criticality equation in this case is

$$\frac{K_\infty e^{-B^2 \tau_0}}{1 + L^2 B^2} = 1 \qquad (9)$$

where $K_\infty = \eta \epsilon p f$, $e^{-B^2 \tau_0}$ is the fast non-leakage probability, and $(1 + L^2 B^2)^{-1}$ is the slow non-leakage probability term. However, the method used to calculate τ_0, (the Fermi Age equation), is not valid in the slowing down region for the case of an hydrogeneous moderator because the assumption of many collisions by the neutrons does not hold. If the moderator contains hydrogen to any degree, it is possible for the neutrons to become thermalized in a few or perhaps even a single collision. Weinberg and Wigner [9], have developed a single collision kernel to account for this type of moderation. The result is that the usual fast non-leakage term is replaced by

$$\frac{\Sigma_0}{B} \tan^{-1}\left(\frac{B}{\Sigma_0}\right) \qquad (10)$$

where Σ_0 is the cross-section for this type of interaction and is given as $\Sigma_0 = \frac{1}{(3\tau_0)^{1/2}}$

where τ_0 is the experimental value of the Fermi Age. Substituting (10) into equation (9) gives

$$\frac{K_\infty \left(\frac{\Sigma_0}{B}\right) \tan^{-1}\left(\frac{B}{\Sigma_0}\right)}{1 + L^2 B^2} = 1$$

This is the form of the modified one-group criticality equation to which strict one-group perturbation theory will be applied.

C. DEVELOPMENT OF PERTURBATION THEORY

In this section, one-group perturbation theory is employed to derive an expression for a change in reactivity from changes in various nuclear parameters.

Let $$C = \frac{\Sigma_o}{B} \tan^{-1}\left(\frac{B}{\Sigma_o}\right)$$

The modified one-group criticality equation for an hydrogenous reactor developed above becomes

$$\frac{K_\infty C}{1 + L^2 B^2} = 1$$

This equation is the direct result of a modified diffusion equation of the following form.

$$\nu \epsilon P C \Sigma_f \phi - \Sigma_a \phi + \text{div } D \text{ grad } \phi = 0 \qquad (11)$$

where $\nu \epsilon P C \Sigma_f \phi$ is the source term, $\Sigma_a \phi$ is the absorption term and div D grad ϕ is the leakage term. Equation (11) can be written as

$$M\phi = 0$$

where M is the operator

$$M = \nu \epsilon P C \Sigma_f - \Sigma_a + \text{div } D \text{ grad}$$

Referring to the discussion of the adjoint operator in Appendix B, the adjoint equation, which will be of use later in the development but which is presented here, is

$$M^+ \psi = 0$$

M+ is the adjoint of M and the function ψ is referred to as the importance function. For one-group perturbation theory, M is self-adjoint and ϕ and ψ are everywhere proportional. Terms in equation (11) which are variables are P, C, Σ_f, Σ_a, and D. Considering first perturbations

or changes in P, C, Σ_f, and Σ_a,

$$P' = P + \delta P$$

$$C' = C + \delta C$$

$$\Sigma_f' = \Sigma_f + \delta\Sigma_f$$

$$\Sigma_a' = \Sigma_a + \delta\Sigma_a$$

The unprimed terms refer to unperturbed quantities, the delta terms are magnitudes of the changes or perturbations, and the primed terms refer to the resulting perturbed quantities. To return the reactor to critical, a change must be made in ν. This is not possible in reality since ν is a physical constant. However, the concept of compensation in this manner is useful in the analysis, so that

$$\nu' = \nu + \delta\nu$$

For the perturbed reactor

$$M'\phi' = 0$$

where

$$M' = (\nu + \delta\nu)(\epsilon)(P + \delta P)(C + \delta C)(\Sigma_f + \delta\Sigma_f) - (\Sigma_a + \delta\Sigma_a) + \text{div } D \text{ grad}$$

Neglecting higher order terms, since for perturbation theory to be valid the perturbations must be small, we get

$$M' = \nu\epsilon PC\Sigma_f + \delta\nu\epsilon PC\Sigma_f + \delta P\nu\epsilon C\Sigma_f + \delta C\nu\epsilon P\Sigma_f + \delta\Sigma_f\nu\epsilon PC - \Sigma_a - \delta\Sigma_a + \text{div } D \text{ grad} \qquad (12)$$

Substituting the quantity M into (12) gives

$$M' = M + \delta\nu\epsilon PC\Sigma_f + \delta P\nu\epsilon C\Sigma_f + \delta\Sigma_f\nu\epsilon PC - \delta\Sigma_a$$

This may also be written as

$$M' = M + A_o$$

where

$$A_o = \delta\nu\epsilon PC\Sigma_f + \delta P\nu\epsilon C\Sigma_f + \delta\Sigma_f\nu\epsilon PC - \delta\Sigma_a$$

Therefore $$(M + A_e)\phi' = 0 \tag{13}$$

$$M^+\psi = 0 \tag{14}$$

Multiplying (13) by ψ, (14) by ϕ', subtracting (14) from (13) and integrating the result over the volume of the fuel core yields

$$\int_v \psi(M + A_o)\phi' dV - \int_v \phi' M^+ \psi \, dV = 0$$

Expanding the first integral,

$$\int_v \psi M\phi' dV + \int_v \psi A_o \phi' dV - \int_v \phi' M^+ \psi \, dV = 0$$

Combining the first and third integrals,

$$\int_v (\psi M\phi' - \phi' M^+ \psi) dV + \int_v \psi A_o \phi' dV = 0 \tag{15}$$

However, from Appendix B, by definition of the adjoint operator,

$$\int_v (\psi M\phi' - \phi' M^+ \psi) dV = 0$$

Therefore, (15) becomes

$$\int_v \psi A_o \phi' dV = 0$$

Substituting the expression for A_o gives

$$\int_v \psi(\delta\nu\epsilon PC\Sigma_f + \delta P\nu\epsilon C\Sigma_f + \delta C\nu\epsilon P\Sigma_f + \delta\Sigma_f\nu\epsilon PC - \delta\Sigma_a)\phi' dV = 0$$

Solving for $\frac{\delta\nu}{\nu}$ and since $\rho = -\frac{\delta\nu}{\nu}$

$$\rho = \frac{\int_v \psi(\delta P\epsilon C\Sigma_f + \delta C\epsilon P\Sigma_f + \delta\Sigma_f\epsilon PC - \frac{\delta\Sigma_a}{\nu})\phi' dV}{\int_v \psi(\epsilon PC\Sigma_f)\phi' dV}$$

Since it is assumed that the perturbation is small,

$$\phi' \approx \phi$$

and because the one-group operator is self-adjoint,

$$\psi \propto \phi$$

and the expression for ρ becomes

$$\rho = \frac{\int_v [\nu(\delta P\epsilon C\Sigma_f + \delta C\epsilon P\Sigma_f + \delta\Sigma_f\epsilon PC) - \delta\Sigma_a]\phi^2 dV}{\int_v (\nu\epsilon PC\Sigma_f)\phi^2 dV} \tag{16}$$

Next, consider a change in D only,

$$D' = D + \delta D$$

Then

$$M' = (\nu + \delta\nu)(\epsilon P C \Sigma_f) + \text{div}\,(D + \delta D)\,\text{grad}$$

Following the analysis previously outlined, the operator for the perturbed system can be written as

$$M' = M + A_o$$

where the perturbation operator is

$$A_o = \delta\nu\epsilon P C \Sigma_f + \text{div}\,\delta D\,\text{grad}$$

This results in a reactivity of

$$\rho = \frac{\int_v \phi\,\text{div}\,\delta D\,\text{grad}\,\phi\,dV}{\int_v (\nu\epsilon P C \Sigma_f)\phi^2 dV} \qquad (17)$$

By employing vector identities,

$$\int_v \phi\,\text{div}\,\delta D\,\text{grad}\,\phi\,dV = \int_v \text{div}\,(\phi\,\delta D\,\text{grad}\,\phi)\,dV - \int_v \delta D(\nabla\phi)^2 dV$$

However, since the flux vanishes at the reactor core surface

$$\int_v \text{div}\,(\phi\,\delta D\,\text{grad}\,\phi)\,dV = \int_A \delta D\,\text{grad}\,\phi \cdot \vec{n}\,dA = 0$$

Hence,

$$\int_v \phi\,\text{div}\,\delta D\,\text{grad}\,\phi\,dV = -\int_v \delta D(\nabla\phi)^2\,dV$$

Therefore, (17) can be expressed as

$$\rho = \frac{\int_v \delta D(\nabla\phi)^2 dV}{\int_v (\nu\epsilon P C \Sigma_f)\phi^2 dV} \qquad (18)$$

Changes in P, C, Σ_f, Σ_a and D occurring simultaneously produce a reactivity represented by the sum of (16) and (18) or

$$\rho = \frac{\int_v \{[\nu(\delta P \epsilon C \Sigma_f + \delta C \epsilon P \Sigma_f + \delta\Sigma_f \epsilon P C) - \delta\Sigma_a]\phi^2 - \delta D(\nabla\phi)^2\}\,dV}{\int_v (\nu\epsilon P C \Sigma_f)\phi^2\,dV}$$

D. APPLICATION OF PERTURBATION THEORY

1. Bare Cylindrical Geometry

In this section the expression derived for the change in reactivity is solved for the case in which the fuel core is unreflected and the actual geometrical dimensions of the core are used. The significant dimensions and coordinate system used are shown in Figure 1. The numerator of the expression for the reactivity contains terms which represent changes in particular reactor parameters in the perturbed system. The perturbation is assumed to occur only in one disk of the core at a time and is uniform within that disk. The total reactivity of the core is found by summing the reactivities of the ten disks. Since the perturbation is zero everywhere in the core outside the disk under consideration, the limits of integration in the numerator are from zero to R in the radial direction and from h to h + δh in the vertical direction. The denominator, however, contains no perturbation terms and is integrated over the entire volume, i.e., from zero to R and from zero to H.

Let

$$I_1 = \int_V \delta\rho\, \nu\epsilon C \Sigma_f \phi^2\, dV$$

$$I_2 = \int_V \delta C\, \nu\epsilon P \Sigma_f \phi^2\, dV$$

$$I_3 = \int_V \delta\Sigma_f\, \nu\epsilon P C \phi^2\, dV$$

$$I_4 = \int_V \delta\Sigma_a \phi^2 dV$$

$$I_5 = \int_V \delta D (\nabla\phi)^2\, dV$$

$$I_6 = \int_V \nu\epsilon P C \Sigma_f \phi^2 dV$$

Therefore,

$$\rho = \frac{I_1 + I_2 + I_3 - I_4 - I_5}{I_6} \qquad (19)$$

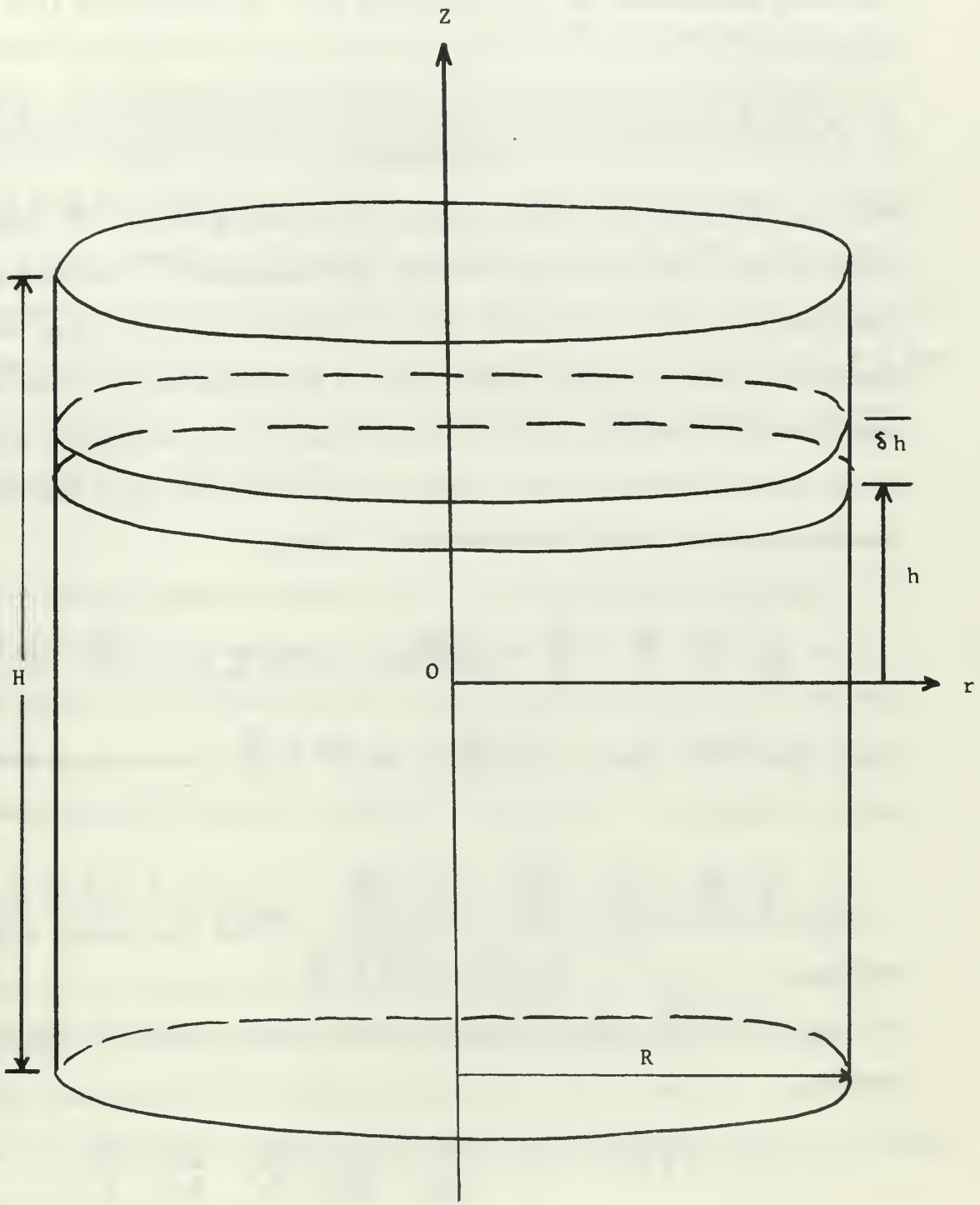

Fig. 1

Fuel Core Dimensions

Evaluation of these I terms is presented in Appendix C. Substituting the resulting expressions for the I terms in (19), and cancelling like terms results in

$$\rho = \frac{\delta\rho\,\nu\epsilon C\Sigma_f Q + \delta C\,\nu\epsilon\rho\Sigma_f Q + \delta\Sigma_f\,\nu\epsilon\rho C Q - \delta\Sigma_a Q - \delta D\left[\alpha^2 Q + \left(\frac{\pi}{H}\right)^2 Y\right]}{\pi\,\nu\epsilon\rho C\Sigma_f}$$

This expression can be further reduced by factoring out the $\frac{Q}{\pi}$ term and dividing each term in the numerator by the denominator. This step is justified only if it is assumed that the values of P, C, Σ_f, Σ_a, and D appearing in the numerator, which refer to disk properties, do not vary significantly from their corresponding values in the denominator which refer to the entire fuel core. Making this assumption makes the final reactivity equation much more tractable. Then,

$$\rho = \frac{Q}{\pi}\left\{\frac{\delta\rho}{\rho} + \frac{\delta C}{C} + \frac{\delta\Sigma_f}{\Sigma_f} - \frac{\delta\Sigma_a}{\nu\epsilon\rho C\Sigma_f} - \frac{1}{\nu\epsilon\rho C\Sigma_f}\left[\alpha^2 + \left(\frac{\pi}{H}\right)^2\frac{Y}{Q}\right]\delta D\right\}$$

Since $K_\infty = \eta\epsilon\rho f$ where $f = \frac{\Sigma_a^F}{\Sigma_f}$ and $\eta = \nu\frac{\Sigma_f}{\Sigma_a^F}$, the expression for ρ can be written as

$$\rho = \frac{Q}{\pi}\left\{\frac{\delta\rho}{\rho} + \frac{\delta C}{C} + \frac{\delta\Sigma_f}{\Sigma_f} - \frac{1}{K_\infty C}\frac{\delta\Sigma_a}{\Sigma_a} - \frac{1}{K_\infty C\Sigma_a}\left[\alpha^2 + \left(\frac{\pi}{H}\right)^2\frac{Y}{Q}\right]\delta D\right\}$$

Letting,

$$e = \alpha^2 + \left(\frac{\pi}{H}\right)^2\frac{Y}{Q}$$

and since $L^2 = \frac{D}{\Sigma_a}$, the final expression for the reactivity for this case is

$$\rho = \frac{Q}{\pi}\left(\frac{\delta\rho}{\rho} + \frac{\delta C}{C} + \frac{\delta\Sigma_f}{\Sigma_f} - \frac{1}{K_\infty C}\frac{\delta\Sigma_a}{\Sigma_a} - \frac{L^2 e}{K_\infty C}\frac{\delta D}{D}\right)$$

2. Extrapolated Dimensions

The derivation of the equation for the reactivity for the case using extrapolated dimensions is similar to that of the bare core geometry. The difference results from the fact that for this analysis the flux is no longer considered to be zero at the physical boundary of the fuel core but instead goes to zero at a hypothetical distance, d, beyond the actual boundary. The resulting flux distribution more accurately represents the actual situation within the reactor core since experimental evidence indicates that the flux is not zero at the core boundary but has a finite value. It can be shown by transport theory that for a planar free surface, d has the value

$$d = 0.71 \ \lambda_{tr}$$

The transport mean free path, λ_{tr} is found from the equation

$$\lambda_{tr} = \frac{\lambda_s}{1 - \bar{\mu}}$$

But since $\bar{\mu}$ is small for the polyethylene moderator, it is assumed that $\lambda_{tr} \approx \lambda_s = \frac{1}{\Sigma_s}$

Substituting this into the equation for d gives

$$d = \frac{0.71}{\Sigma_s}$$

For a curved free surface, such as the outer edge of the cylindrical fuel core, the above equation for d can still be used and introduces negligible error provided that d is small compared to the radius R. This condition will be shown to be true for this reactor. Therefore, because of the extrapolation distance, the extrapolated radius and extrapolated height of the fuel core become

$$R' = R + d$$

$$H' = H + 2d$$

and the parameters α and β in the flux expression become

$$\alpha = \frac{2.405}{R'} \qquad \beta = \frac{\pi}{H'}$$

From this point on, the development of the reactivity equation follows that outlined in the preceding section with two exceptions. First, several of the zero order Bessel functions in the reactivity equation for the bare core geometry case were zero because of the boundary condition that the flux equal zero at R. When using the extrapolated radius R' these Bessel functions are not equal to zero and are retained in the equation. Secondly, using the extrapolated height H', rather than H in many of the integrals involving trigonometric functions yields a slightly different result. Because much of the development of the reactivity equation is repetition of the bare core geometry case it will not be repeated here, but it can be shown that for this case that

$$\rho = \frac{T}{2A}\left(\frac{\delta\rho}{\rho} + \frac{\delta C}{C} + \frac{\delta\Sigma_f}{\Sigma_f} - \frac{1}{K_\infty C}\frac{\delta\Sigma_a}{\Sigma_a} - \frac{L^2 b}{K_\infty C}\frac{\delta D}{D}\right)$$

where

$$T = \frac{\pi\delta h}{H'} + \frac{1}{2}\sin\frac{2\pi(h+\delta h)}{H'} - \frac{1}{2}\sin\frac{2\pi h}{H'}$$

$$A = \frac{\pi H}{2H'} + \frac{1}{4}\sin\frac{2\pi H}{H'}$$

$$b = \frac{X\alpha^2}{E} + \left(\frac{\pi}{H'}\right)\frac{W}{T}$$

$$W = \frac{\pi\delta h}{H'} + \frac{1}{2}\sin\frac{2\pi h}{H'} - \frac{1}{2}\sin\frac{2\pi(h+\delta h)}{H'}$$

$$X = J_1(\alpha R) - J_0(\alpha R)J_2(\alpha R)$$

and

$$E = J_0^2(\alpha R) + J_1^2(\alpha R)$$

3. Reflected Core

Since the fuel core of the AGN-201 reactor is surrounded by a graphite reflector, the expression derived for the reactivity in this section probably is the most realistic and accurate of the three cases discussed, even if it is only an approximate solution. As it turns out, the problem of a cylindrical core with radial and end reflectors, as is the case for this reactor, is more than can be handled in an exact analytical manner. The problem arises from attempting to find a form in which to represent the flux distribution so as to satisfy the boundary conditions at the "corners" of the cylinder, i.e., at r = R and z = H/2. The problem can be solved by numerical methods using relaxation techniques but is rather complicated and will not be used here.

An approximate method of solution is to treat the reactor as having a radial reflector and bare ends but having a height of $H + 2\delta_z$ where δ_z is the end reflector savings given by

$$\delta_z = \frac{1}{\beta} \tan^{-1}\left(\frac{D_c \beta L_r}{D_r} \tanh \frac{T_z}{L_r}\right)$$

Therefore in the flux distribution expression,

$$\beta = \frac{\pi}{H + 2\delta_z}$$

Since the reactor core is considered to have only a radial reflector

$$\alpha = \frac{2.405}{R + \delta_r}$$

where

$$\delta_r = \frac{1}{\beta} \tan^{-1}\left(\frac{D_c \beta L_r}{D_r} \tanh \frac{T_r}{L_r}\right)$$

From Appendix A it can be noted that the thickness of the reflector is uniform about the reactor core so that

$$T_r = T_z = T$$

From this, it follows that

$$\delta_r = \delta_z = \delta$$

Following this preliminary discussion, the development of the reactivity

expression follows closely that of the extrapolated dimensions case, only consideration is directed toward the effect of the reflector savings concept rather than the extrapolation distance. The resulting expression for the reactivity for this case is

$$\rho = \frac{F}{2G}\left(\frac{\delta P}{P} + \frac{\delta C}{C} + \frac{\delta \Sigma_f}{\Sigma_f} - \frac{1}{K_\infty C}\frac{\delta \Sigma_a}{\Sigma_a} - \frac{L^2 a}{K_\infty C}\frac{\delta D}{D}\right)$$

where

$$F = \frac{\pi \delta h}{H + 2\delta} + \frac{1}{2}\sin\frac{2\pi(h+\delta h)}{H+2\delta} - \frac{1}{2}\sin\frac{2\pi H}{H+2\delta}$$

$$G = \frac{\pi H}{2(H+2\delta)} + \frac{1}{4}\sin\frac{2\pi H}{H+2\delta}$$

$$a = \frac{X\alpha^2}{E} + \left(\frac{\pi}{H+2\delta}\right)^2 \frac{S}{F}$$

and

$$S = \frac{\pi}{H+2\delta} + \frac{1}{2}\sin\frac{2\pi h}{H+2\delta} - \frac{1}{2}\sin\frac{2\pi(h+\delta h)}{H+2\delta}$$

IV. COMPUTER ANALYSIS

A. DERIVATION OF NUMERICAL COEFFICIENTS

In the preceding sections, equations for the change in reactivity of the perturbed reactor system for three different models were developed. These equations were:

Bare core geometry,

$$\rho = \frac{Q}{\pi}\left(\frac{\delta P}{P} + \frac{\delta C}{C} + \frac{\delta \Sigma_f}{\Sigma_f} - \frac{1}{K_\infty C}\frac{\delta \Sigma_a}{\Sigma_a} - \frac{L^2 e}{K_\infty C}\frac{\delta D}{D}\right) \tag{20}$$

Extrapolated dimensions,

$$\rho = \frac{T}{2A}\left(\frac{\delta P}{P} + \frac{\delta C}{C} + \frac{\delta \Sigma_f}{\Sigma_f} - \frac{1}{K_\infty C}\frac{\delta \Sigma_a}{\Sigma_a} - \frac{L^2 b}{K_\infty C}\frac{\delta D}{D}\right) \tag{21}$$

Reflected core,

$$\rho = \frac{F}{2G}\left(\frac{\delta P}{P} + \frac{\delta C}{C} + \frac{\delta \Sigma_f}{\Sigma_f} - \frac{1}{K_\infty C}\frac{\delta \Sigma_a}{\Sigma_a} - \frac{L^2 a}{K_\infty C}\frac{\delta D}{D}\right) \tag{22}$$

As can be seen, each of the equations contains the terms P, C, Σ_f, K_∞, Σ_a, L^2, and D. These parameters refer to the cylindrical fuel core as a whole, are constants, and are evaluated at their initial values, prior to the reactivity drop. From Appendix D, where expressions for these parameters are developed and numerically evaluated using data from the AGN-201, the following results are obtained.

$$P = 0.9473$$

$$C = 0.7414$$

$$\Sigma_f = 0.06507 \text{ cm}^{-1}$$

$$K_\infty = 1.510$$

$$\Sigma_a = 0.1010 \text{ cm}^{-1}$$

$$L^2 = 2.177 \text{ cm}^2$$

$$D = 0.2199 \text{ cm}$$

Before these reactivity equations describing the three models can be used, a discussion of the term δC must be presented. Recalling that

$$C = \frac{\Sigma_0}{B} \tan^{-1}\left(\frac{B}{\Sigma_0}\right)$$

where B is a constant and

$$\Sigma_0 = \frac{1}{(3\tau_0)^{1/2}}$$

the problem is to determine the manner in which τ_0 varies as a function of the moderator density. The Fermi Age, τ_0, is proportional to the inverse of the product of the logarithmic energy loss per collision, $\bar{\xi}$, and the macroscopic scattering cross-section, Σ_s.

$$\tau_0 \propto \frac{1}{\bar{\xi}\Sigma_s}$$

To make this relation an equality, let the constant of proportionality be ϵ_T, so that

$$\tau_0 = \frac{\epsilon_T}{\bar{\xi}\Sigma_s}$$

From Appendix D, $\tau_0 = 8.459 \text{ cm}^2$

$\bar{\xi} = 0.9090$

and $\Sigma_s = 1.516 \text{ cm}^{-1}$

Therefore, $\epsilon_T = 17.75$

Since Σ_s is a function of the moderator density, the relation between τ_0 and hence C, and ρ is

$$\tau_0 = \frac{17.75}{\bar{\xi}\Sigma_s}$$

1. Bare Cylindrical Geometry

The term e in the bare core geometry equation (20) has been defined as

$$e = \alpha^2 + \left(\frac{\pi}{H}\right)^2 \frac{Y}{Q}$$

From Appendix D, $\alpha^2 = 0.03587 \text{ cm}^{-2}$

and $H = 12.7$ cm

Therefore $$e = 0.03587 + 0.1695 \frac{Y}{Q} \qquad (23)$$

Substitution of the values for the core parameters and (23) into equation (20) yields the numerical equation for the reactivity for the bare core geometry model, which is

$$\rho = \frac{Q}{\pi}\left[(1.056)\delta P + (1.349)\delta C + (15.37)\delta\Sigma_f - (8.844)\delta\Sigma_a - (0.3172 + 0.1499\frac{Y}{Q})\delta D\right] \qquad (24)$$

2. Extrapolated Dimensions

In the equation for reactivity in the extrapolated dimensions model, (21), b represents the quantity

$$b = \frac{X\alpha^2}{E} + \left(\frac{\pi}{H'}\right)^2 \frac{W}{T} \qquad (25)$$

From previous analysis it has been assumed that d, the extrapolation distance, can be calculated from the equation

$$d = \frac{0.71}{\Sigma_s}$$

From Appendix D, $\Sigma_s = 1.516\ cm^{-1}$

so that $d = 0.47$ cm

The extrapolated dimensions are therefore

$$R' = R + d = 13.17 \text{ cm}$$

$$H' = H + 2d = 25.07 \text{ cm}$$

In equation (25), for the extrapolated dimensions case

$$\alpha = \frac{2.405}{R'} = 0.1826\ cm^{-1}$$

$$X = J_1(\alpha R) - J_0(\alpha R)\, J_2(\alpha R)$$

and $$E = J_0^2(\alpha R) + J_1^2(\alpha R)$$

From a table of Bessel functions, and since $\alpha R = 2.319$,

$$J_0(\alpha R) = 0.04531$$

$$J_1(\alpha R) = 0.5364$$

$$J_2(\alpha R) = 0.4173$$

So that $X = 0.5175$

and $E = 0.2898$

Substituting these values into (25) yields

$$b = 0.05954 + 0.01631 \frac{W}{T} \qquad (26)$$

Inserting the values for the core parameters and the expression for b, (26), into equation (21), gives the numerical equation for the reactivity in the extrapolated dimensions model as

$$\rho = \frac{T}{2A}\left[(1.056)\delta P + (1.349)\delta C + (15.37)\delta\Sigma_f - (8.844)\delta\Sigma_a - (0.5265 + 0.1442\frac{W}{T})\delta D\right] \qquad (27)$$

3. Reflected Core

To evaluate the numerical coefficients in the reactivity equation for the reflected core model, (22), the first step is to evaluate the reflector savings term, δ .

$$\delta = \frac{1}{B}\tan^{-1}\left(\frac{DBL_r}{D_r}\tanh\frac{T}{L_r}\right) \qquad (28)$$

From Appendix A, $T = 20$ cm

The values for B and D have already been given as

$$B = 0.2298 \text{ cm}^{-1}$$

$$D = 0.2199 \text{ cm}$$

Since the reflector material is high grade graphite,

$$L_r = 54.2 \text{ cm}$$

$$D_r = 0.94 \text{ cm}$$

Therefore, evaluation of (26) yields

$$\delta = 3.49 \text{ cm}$$

so that $R + \delta = 16.19$ cm

$H + 2\delta = 31.11$ cm

For the reflected case, the parameter α in the expression for the flux distribution has been given as

$$\alpha = \frac{2.405}{R+\delta} = 0.1485 \text{ cm}^{-1}$$

so that $\alpha R = 1.887$

Since $X = J_1(\alpha R) - J_0(\alpha R) J_2(\alpha R)$

and $E = J_0^2(\alpha R) + J_1^2(\alpha R)$

Substitution of the values for the Bessel functions of

$J_0(\alpha R) = 0.2898$

$J_1(\alpha R) = 0.5814$

$J_2(\alpha R) = 0.3269$

yields $X = 0.4868$

$E = 0.4218$

Substitution in the expression

$$a = \frac{X\alpha^2}{E} + \left(\frac{\pi}{H+2\delta}\right)^2 \frac{S}{F}$$

gives an expression for a of

$$a = 0.02597 + 0.01020 \frac{S}{F} \qquad (29)$$

Putting this expression, (29), and the values of the core parameters into (22) gives

$$\delta = \frac{F}{2G}\left[(1.056)\delta P + (1.349)\delta C + (15.37)\delta\Sigma_f - (8.844)\delta\Sigma_a - (0.2297 + 0.09020\frac{S}{F})\delta D\right] \qquad (30)$$

as the numerical equation for the reactivity in the reflected core model.

B. FLUX WEIGHTING OF CHANGE IN MODERATOR

It will be noted that four-figure accuracy is maintained in the evaluation of all the above parameters. This is not strictly justified on the basis of the accuracy of the data available on the AGN-201 reactor, but was made necessary by the very small magnitude of the reactivity change, 0.00073, which was to be accounted for. The validity of the above numerical results is strengthened somewhat by using a weighted distribution of the change in density within the fuel core. It seems almost certain that the change in hydrogen density is not uniform within the fuel core, but is instead higher at the center, where the neutron flux is highest and decreases towards the edges of the core. While the density change was assumed uniform within a single disk within the core, the distribution of the density change in the vertical direction was "weighted" by a cosine function corresponding to the shape of the flux distribution in the vertical direction.

Define a weighting function = $\cos \frac{\pi z}{H}$

Using the distance of the centerline of each disk (z_d) from the zero ordinate of the z-axis of the fuel core, the fraction of the total density change within a single disk Δ_n, can be found from the expression

$$\Delta_n = \frac{\cos \frac{\pi Z_{d_n}}{H}}{\sum_{n=1}^{10} \cos \frac{\pi Z_{d_n}}{H}}$$

The results of applying this equation to the data listed in column two of Table 1 is shown in column four of that same table.

TABLE 1

Disk	z_d	$\cos \frac{\pi z_d}{H}$	Δ
1	4.66	0.02973	0.00584
2	4.37	0.1253	0.02464
3	4.00	0.2455	0.04828
4	3.44	0.4198	0.08256
5	2.68	0.6323	0.1243
6	1.94	0.8012	0.1576
7	0.83	0.9626	0.1893
8	-1.00	0.9458	0.1860
9	-2.50	0.6773	0.1332
10	-4.00	0.2455	0.04828

C. COMPUTER PROGRAMS

All of the discussion in this section to this point has served to establish the basis for the final application of perturbation theory to the analysis of the behavior of the AGN-201 reactor. Appendix E contains a digital computer program, the results of which demonstrate the reactivity change of the reactor as a function of moderator density change caused by operation of the reactor. In the program, the change in moderator mass starts at 10 gm and increases to 50 gm, in increments of 10 gm. This overall change in mass is weighted according to the results listed in Table 1, to find the density change per disk. For each change in moderator mass, the program uses the reactivity equations (24), (27), and (30) to calculate the reactivity drop per disk and for the entire core

for each of the three models describing the reactor core. The numerical results of this program are presented formally in the next section. However, at this point, it is sufficient to state that the density change predicted by the perturbation theory equations corresponding to the actual value of the reactivity drop equal to 0.073% are as follows:

Bare core geometry: ΔM_ρ = 17.9 gm

Extrapolated dimensions: ΔM_ρ = 15.1 gms

Reflected core: ΔM_ρ = 13.8 gm

These values for the total core mass change are used in the computer program presented in Appendix F, where the effect of adding a new fuel disk of known properties is analyzed. Using the data for the new fuel disk listed in Appendix A and the results of the first computer program to determine the present properties of the old disks currently in the reactor, the resulting excess reactivity of the reactor is predicted when the new disk displaces a disk of similar size by again using the numerical perturbation equations (24), (27), and (30). The results of this program are presented and discussed in the following section.

V. RESULTS AND CONCLUSIONS

The numerical results of Computer Program 1 presented in Appendix E are listed in Appendix G and plotted graphically in Figure 2. It appears from Figure 2 that a linear relationship exists between the reactivity of the reactor core and the change in moderator density within the core. Taking the slopes of these plots for each of the three models shown, the following expressions can be derived:

Bare core geometry: $\Delta\rho = -0.00004095\ M_p$

Extrapolated dimensions: $\Delta\rho = -0.00004888\ M_p$

Reflected core: $\Delta\rho = -0.00005302\ M_p$

Also, from Figure 2 it is possible to obtain the change in polyethylene mass for each model which corresponds to the observed change in reactivity of the reactor of 0.00073. These changes are

Bare core geometry: $M_p = 17.9$ gms

Extrapolated dimensions: $M_p = 15.1$ gms

Reflected core: $M_p = 13.8$ gms

In previous discussion a value of the weight of hydrogen bled off from the reactor during its lifetime of operation was estimated as 23.14 gm. While this value is slightly higher than that predicted by the perturbation theory equations, no true error analysis can be conducted. The value of 23.14 is actually only a crude approximation in itself, but the fact that this experimental value and the theoretical values are reasonably close to one another lends validity to the results of this analysis. It can also be stated that the average flux level of 20 watts used to calculate the value of 23.14 gm must be close to the actual value of the reactor flux. Since this is the case, the arguments discounting fuel

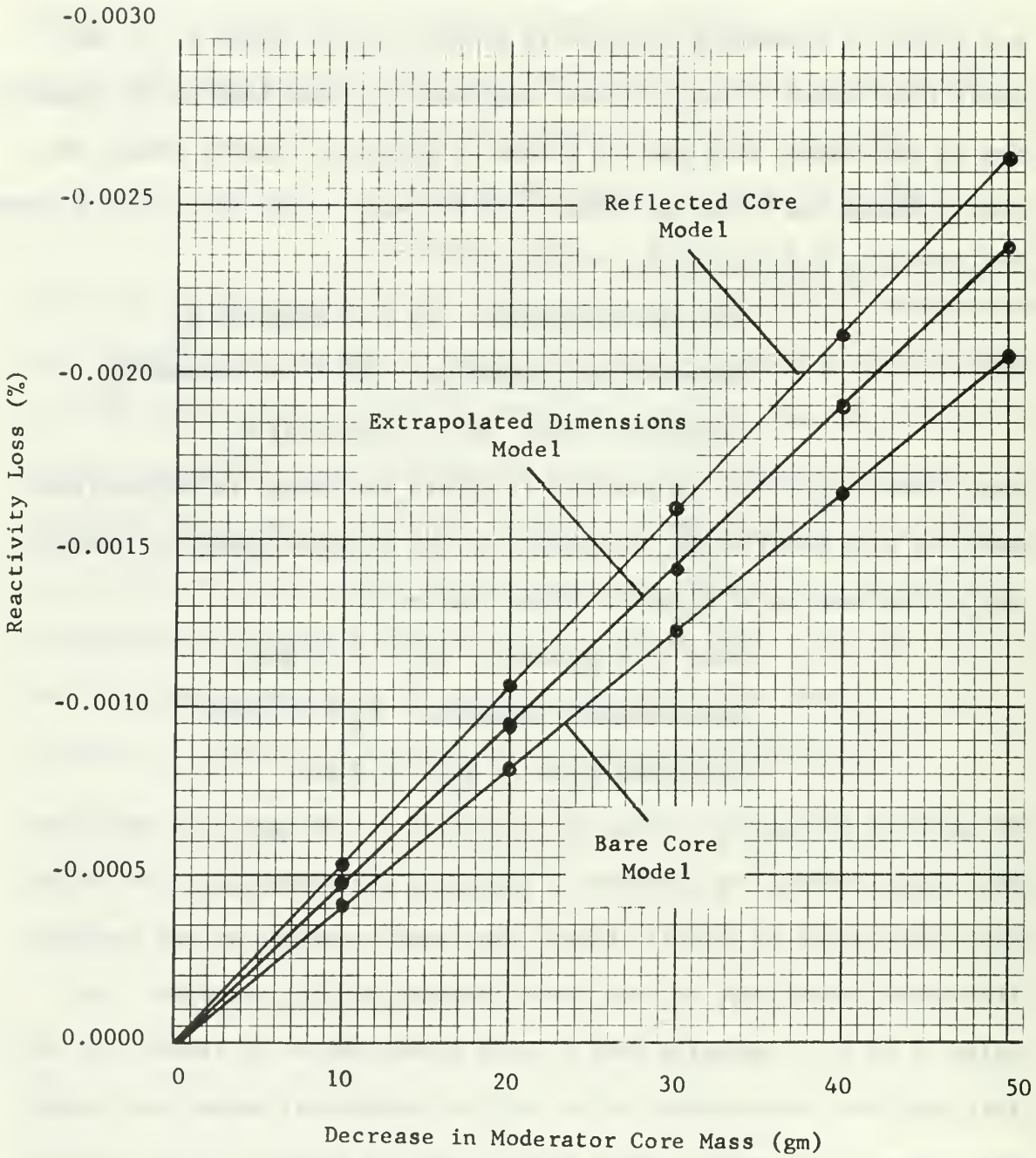

Fig. 2

Reactivity Loss versus Decrease in Moderator Core Mass

burnup and reactor poisons as sources of reactivity drop are reinforced.

Interpolation of the data in Tables 6 and 7 of Appendix G to correspond to the above changes in moderator mass allows an analysis to be made of the reactivity drop in the reactor core as a function of vertical position. These results are plotted in Figure 3. As can be seen, the curves for the three models are different. The extrapolated dimensions model curve, in general, shows a higher overall value for the reactivity decrease than the bare core model curve. The difference noted between the reflected core model curve and the extrapolated dimensions model curve is that the reflected core curve is slightly "flattened". The reactivity decrease is higher at the top and bottom of the core and lower in the middle for the reflected core. The variations among the three curves reflect the various shapes of the assumed flux within the core for each model.

Using the density changes for the three models of 17.9, 15.1, and 13.8 gm to determine the present core density of the reactor, the results of Computer Program 2 in Appendix F show that the effect of interchanging the new disk with an old disk of the same size, (either disk 4,5, or 6), results in a reactivity increase which exceeds the licensed limiting value of 0.40%. This can be confirmed by referring to Table 2 or Appendix H. The large reactivity increase can be attributed to the large increase in moderator density between the old disks and the new disk. This increase in density is approximately 0.2 gm/cm^3 higher for the new disk. By modifying Computer Program 2 to print out the values of δC and δD, it was found that these two positive terms in the reactivity equation, (δD is negative, but is multiplied by a negative coefficient) which are functions of moderator density, increase substantially

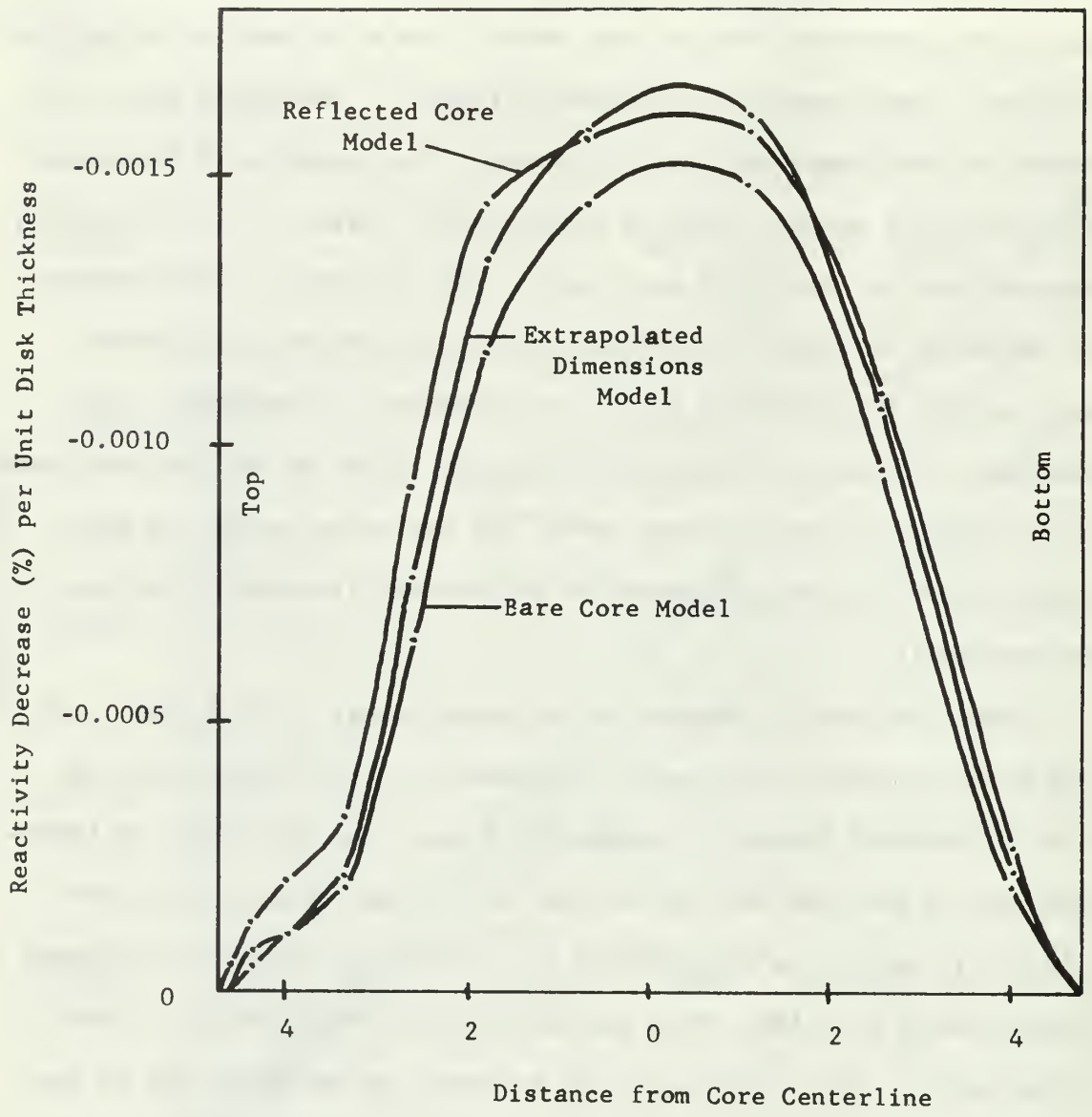

Fig. 3

Spatial Variation of Reactivity Within Core

REACTIVITY

MODEL / DISK	Bare Core	Extrapolated Dimensions	Reflected Core
4 (Z_d=3.44)	0.004392	0.004816	0.005767
5 (Z_d=2.68)	0.005300	0.005938	0.006643
6 (Z_d=1.94)	0.006481	0.007351	0.007707

TABLE 2

when the new disk displaces an old disk. These increases more than compensate for the negative contribution to reactivity caused by the decrease in fuel density of the new disk as compared to the old disks. For purposes of illustration, consider the values obtained from Computer Program 2 listed in Table 3. These values are those obtained if the new disk were to displace Disk 4 using the reflected core model.

TABLE 3

$\delta P = 0.006269$

$\delta C = 0.04459$

$\delta\sigma_f = 4.931$ b

$\delta\Sigma_f = -0.00003234$ cm^{-1}

$\delta\sigma_a^{235} = 5.892$ b

$\delta\sigma_a^{238} = 0.02357$ b

$\delta\Sigma_a^{P} = 0.004563$ cm^{-1}

$\delta\Sigma_a = 0.004523$ cm^{-1}

$\delta D = -0.02916$ cm

From this table it can be seen that the changes in parameters caused directly by moderator density changes, when multiplied by their corresponding coefficients in the reactivity equation, are of about the same order of magnitude as those caused indirectly by the change in cross-section caused by the change in neutron temperature discussed earlier. Thus, it is concluded that a simple interchange of the new fuel disk with any one of three similar size disks which are presently in the reactor core will produce too large a value for the excess reactivity.

The conclusions drawn from this perturbation theory analysis of reactivity may be itemized as follows:

1) The decrease in reactivity due to loss of moderator within the core is a linear function of the change in core density.

2) Due to operation of the AGN-201 reactor over a period of several years, the moderator mass has decreased from between 14 to 18 gms.

3) The interchange of the new fuel disk with an old disk of similar size currently in the reactor and in its present position within the core will produce an excess reactivity exceeding a value of 0.40%.

VI. RECOMMENDATIONS

There are several areas into which further study of the problem of the reactivity analysis of the AGN-201 reactor could be directed. Based on the results obtained in this thesis, a direct displacement of an old disk of the same size by the new disk would exceed the limit of 0.40% excess reactivity. The perturbation equations developed here could be used, however, to predict the change in reactivity resulting from any new combination of disks in the core, such as displacing an old disk with the new disk but removing the smallest top disk, adding the new disk at the top of the core, etc. Only slight modification of the data input into the computer programs would be necessary to permit this. In other words, the equations and computer programs developed here could be used to find a disk arrangement such that the excess reactivity approached 0.40%.

Secondly, a chemical analysis could be conducted both to determine the exact products formed after polyethylene is irradiated in the core and the nature of products present in the evolved gases. In this thesis the assumption was made that only hydrogen was evolved and that no new type of molecule was formed in the fuel core. Perhaps a chemical analysis of irradiated polyethylene could either substantiate this or illuminate more clearly the exact nature of this phenonemon.

Third, the method used to relate neutron temperature to moderator density, equation (8), was only an empirical relation which was modified to suit the data in this thesis. A more exact method of handling this problem is to treat the problem by an energy-dependent model, with the scattering kernel computed by the code GAKER, which is available through such agencies as the Argonne National Laboratory. The use of such a code,

if obtained and programmed to run on the IBM-360 computer would undoubtedly increase the accuracy of the results of this type of analysis.

Finally, other analytical improvements could be made in this analysis. The results of a chemical analysis, such as previously described, might indicate that a two-group analysis rather than one-group as used here, might provide better results. Also, a more exact analytical method of handling the reflected core model would be desirable.

APPENDIX A

DESCRIPTION OF THE REACTOR

A. GENERAL FEATURES:

The AGN-201 reactor is a low power nuclear reactor designed primarily for research and educational purposes. It has a fuel core consisting of ten uranium impregnated polyethylene circular disks 10 inches in diameter and varying in thickness from 3/16 inches to 1-1/2 inches, stacked on top of one another to form a cylindrical core. The uranium is in the form of uranium dioxide enriched to 19.93% with U-235 and is uniformly dispersed in the polyethylene. Inserted in the core through its base are four control rods. Three are the same size; the fourth is smaller. Two of the larger rods serve as safety rods, the third as a coarse control rod, and the smaller rod functions as a fine control rod. Since these rods are of the same material as the core, their contributions to reactivity considerations are handled by incorporating their properties into the disk of which they form a part when fully inserted into the core. This is both justified and necessary when calculating the reactivity increase caused by adding a new fuel disk since insertion of the rods increases reactivity and the maximum excess reactivity value is not to be exceeded.

Surrounding the fuel core on all sides is a reflector of high density (1.7 gms-cm^{-3}) graphite, 20 cm in thickness. Shielding is provided by a 10 cm thick lead shield, a thick steel tank having a radius of 43.4 cm, a borated-water-filled shield tank 6-1/2 feet in diameter, and finally, a housing of large concrete blocks which encloses the entire reactor to

provide protection against radiation while operating at higher power levels. The general features of the AGN-201 reactor are shown in Figure 4. The core tank of the reactor is fluid tight and leak tested to a pressure of 5 psig. To handle pressure build-ups during operation and after shut-down, the tank is connected with copper tubing to a gas handling system which provides a means for measuring core pressure and a means for bleeding, storing, and monitoring the gases generated by the fuel disks. To insure against ignition of the hydrogen gas with any oxygen in the air which might have been initially in the fuel core, a system feeding nitrogen, an inert gas, into the core tank is provided.

B. THE FUEL CORE:

The dimensions of the fuel core and of the ten disks composing it are shown schematically in Figure 5. For convenience of reference, the disks have been arbitrarily assigned numbers. Pertinent disk and core data is listed in Table 4. It should be noted that while the various physical properties are constant within a single disk, variations exist from disk to disk.

C. NEW FUEL DISK

Data pertaining to the new fuel disk to be inserted in the fuel core of the reactor is presented in Table 5.

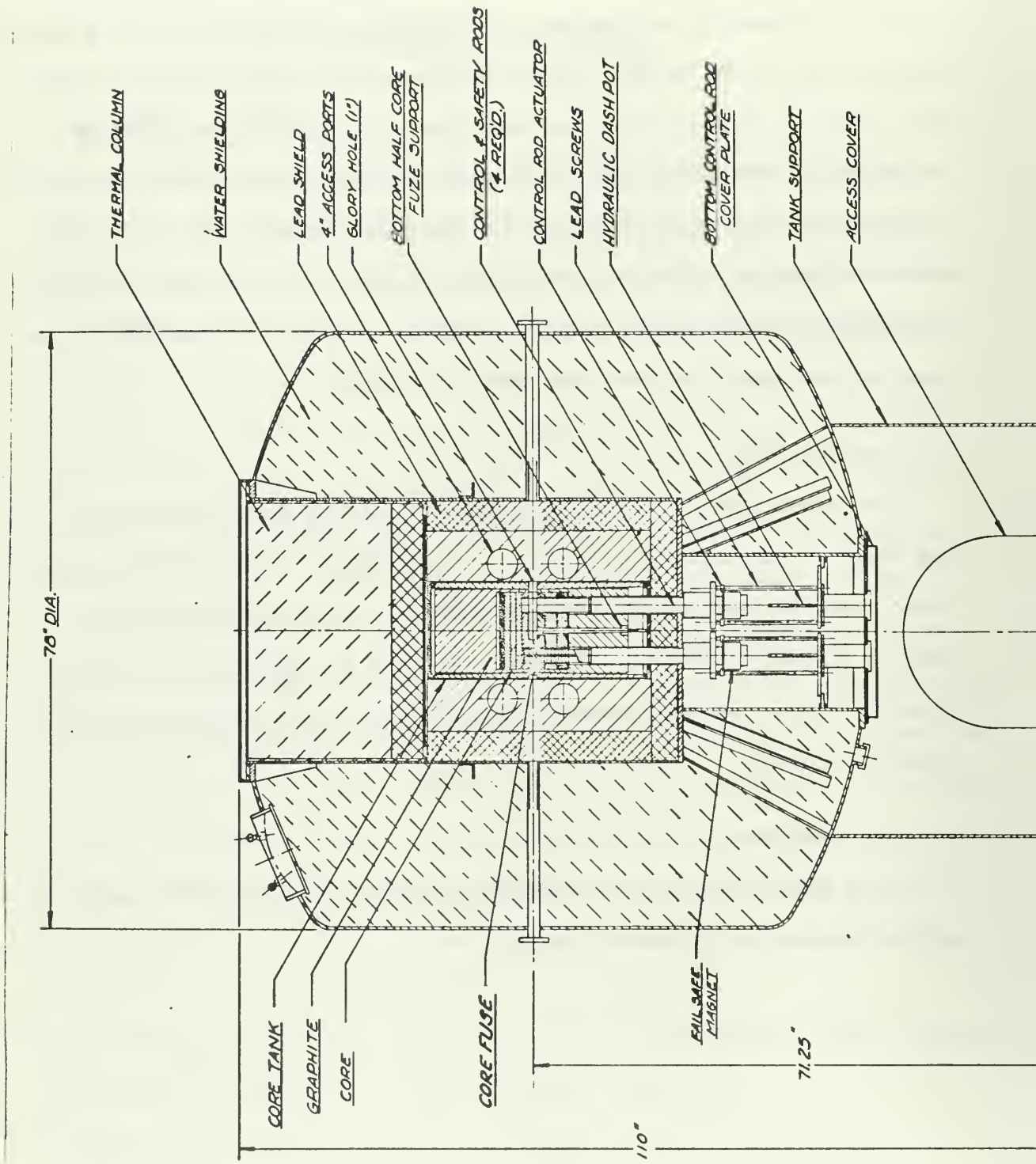

Fig. 4

The AGN-201 Reactor

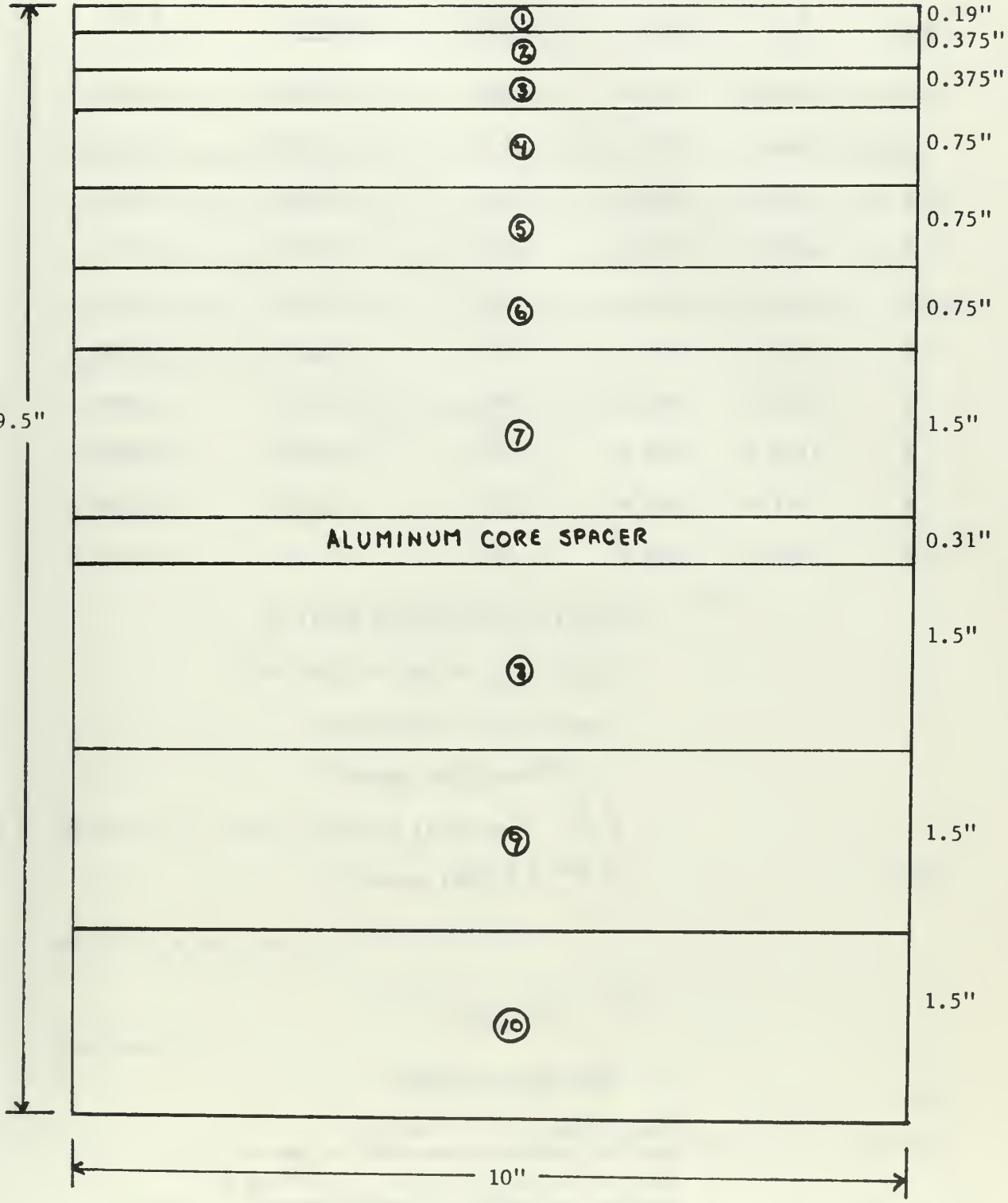

Fig. 5

Fuel Core

TABLE 4

Fuel Core and Disk Data

Disk	M_p	V_d	ρ^p	ρ^{235}	ρ^{238}
1	292.5	224.6	1.196	0.06958	0.2796
2	498.2	482.7	1.032	0.05625	0.2449
3	552.4	482.7	1.144	0.06648	0.2670
4	1038.1	965.4	1.075	0.06259	0.2515
5	1049.1	965.4	1.087	0.06313	0.2536
6	1047.2	965.4	1.085	0.06303	0.2532
7	1721.2	1930.8	0.8914	0.05179	0.2080
8	1722.9	1930.8	0.8923	0.05190	0.2084
9	1761.8	1930.8	0.9125	0.05298	0.2129
10	1784.8	1930.8	0.9244	0.05371	0.2157

Total core weight = 14821 gm

Total core volume = 12230 cm^3

Overall core densities:

$\rho^p = 0.9383\ gm\text{-}cm^{-3}$

$\rho^{235} = 0.05451\ gm\text{-}cm^{-3}$

$\rho^{238} = 0.2191\ gm\text{-}cm^{-3}$

$\rho^u = 0.2736\ gm\text{-}cm^{-3}$

TABLE 5

New Fuel Disk Data

Disk volume = 965.4 $gm\text{-}cm^{-3}$
Mass of polyethylene = 921.4 gm
Density of polyethylene = 1.268 $gm\text{-}cm^{-3}$
Density of U-235 = 0.06003 $gm\text{-}cm^{-3}$
Density of U-238 = 0.2287 $gm\text{-}cm^{-3}$

APPENDIX B

DISCUSSION OF THE ADJOINT OPERATOR

The fuel core of the AGN-201 reactor is a finite cylinder, and the diffusion equation governing its behavior is a partial differential equation. While the following discussion applies equally well to such a case, for simplicity of presentation, consider the following differential equation which is the one -group diffusion equation for a critical slab reactor shown below.

$$\frac{d}{dx} D(x) \frac{d\phi}{dx} + F(x) = 0$$

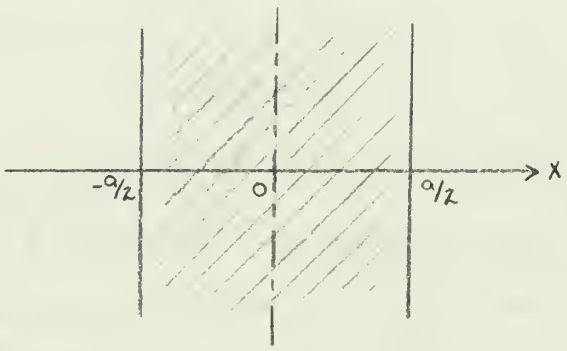

Fig. 6

This can be written as

$$M\phi = 0 \qquad (31)$$

where M is the operator

$$M = \frac{d}{dx} D(x) \frac{d}{dx} + F(x)$$

The equation

$$M^{+}\psi = 0 \qquad (32)$$

is related to equation (31) and is referred to as the adjoint equation. M^{+} is known as the adjoint of M and the function ψ is referred to as the importance function. In addition to satisfying equation (32), ψ

must satisfy the same boundary conditions as ϕ, which for this case is to vanish at $X = \pm a/2$, the slab half-thickness. The adjoint operator M+ is defined as that operator which satisfies the equality,

$$\int_{-a/2}^{a/2} v M w \, dx = \int_{-a/2}^{a/2} w M^{+} v \, dx \tag{33}$$

where v and w are any two functions which vanish at $X = \pm a/2$. Substituting the expression for M into the left-hand side of (33) and integrating by parts yields

$$\int_{-a/2}^{a/2} v\left[\frac{d}{dx} D \frac{d}{dx} + F\right] w \, dx = \left(vDw' - v'Dw\right)\Big|_{-a/2}^{a/2} + \int_{-a/2}^{a/2} w\left[\frac{d}{dx} D \frac{d}{dx} + F\right] v \, dx \tag{34}$$

But since $v(\pm a/2) = 0$

and $w(\pm a/2) = 0$

then, $\left(vDw' - v'Dw\right)\Big|_{-a/2}^{a/2} = 0$

and from (34)

$$\int_{-a/2}^{a/2} v M w \, dx = \int_{-a/2}^{a/2} w M v \, dx \tag{35}$$

Comparison of (35) with (33) shows that for one-group theory

$$M+ = M$$

When an operator and its adjoint are identical, the operator is termed self-adjoint. Further, because equations (31) and (32) are homogeneous, and ψ and ϕ satisfy the same boundary conditions, ϕ and ψ are everywhere proportional. [5]

APPENDIX C

EVALUATION OF INTEGRAL TERMS IN REACTIVITY EQUATION

Evaluation of the I terms in the reactivity equation (19) is as follows:

$$I_1 = \int_v \delta P \nu \epsilon C \Sigma_f \phi^2 \, dV$$

For a cylinder,

$$dV = 2\pi r \, dr \, dZ$$

and the flux distribution, ϕ , is

$$\phi = \phi_0 J_0(\alpha r) \cos \beta Z$$

where

$$\alpha = \frac{2.405}{R}$$

and

$$\beta = \frac{\pi}{H}$$

Therefore

$$I_1 = \int_0^R \int_h^{h+\delta h} \delta P \nu \epsilon C \Sigma_f \phi_0^2 J_0^2(\alpha r) \cos^2(\beta Z)(2\pi r \, dr \, dZ)$$

Since ν , ϵ , C, Σ_f, and ϕ_0 are constants within a disk and δP, the perturbed parameter, is a function only of a density change which is assumed uniform within a single disk, I_1 can be written as

$$I_1 = 2\pi \delta P \nu \epsilon C \Sigma_f \phi_0^2 \int_0^R r J_0^2(\alpha r) dr \int_h^{h+\delta h} \cos^2(\beta Z) \, dZ$$

Evaluating,

$$\int_0^R r J_0^2(\alpha r) \, dr = \frac{R^2}{2} J_1^2(\alpha R)$$

since

$$J_0^2(\alpha R) = J_0^2\left(\frac{2.405 R}{R}\right) = 0$$

$$\int_h^{h+\delta h} \cos^2(\beta Z) \, dZ = \frac{HQ}{2\pi}$$

where

$$Q = \frac{\pi \delta h}{H} + \frac{1}{2} \sin \frac{2\pi(h+\delta h)}{H} - \frac{1}{2} \sin \frac{2\pi h}{H}$$

Therefore,

$$I_1 = \frac{1}{2} \delta P \nu \epsilon C \Sigma_f \phi_0^2 R^2 J_1^2(\alpha R) HQ$$

Through a similar analysis it is found that

$$I_2 = \tfrac{1}{2}\,\delta C \upsilon \epsilon P \Sigma_f \phi_o^2 R^2 J_1^2(\alpha R)\, HQ$$

$$I_3 = \tfrac{1}{2}\,\delta \Sigma_f \upsilon \epsilon P C \phi_o^2 R^2 J_1^2(\alpha R)\, HQ$$

and $$I_4 = \tfrac{1}{2}\,\delta \Sigma_a \phi_o^2 R^2 J_1^2(\alpha R)\, HQ$$

The solution for the I_5 term is somewhat different since the diffusion coefficient is weighted by the gradient of the flux rather than by the square of the flux. The gradient of the flux can be written as

$$\nabla\phi = \vec{u}_r \frac{\partial\phi}{\partial r} + \vec{u}_\theta \frac{1}{r}\frac{\partial\phi}{\partial\theta} + u_z \frac{\partial\phi}{\partial z}$$

But, $\vec{u}_\theta \frac{1}{r}\frac{\partial\phi}{\partial\theta} = 0$, since it is assumed $\phi \neq \phi(\theta)$

Therefore, $$(\nabla\phi)^2 = \left(\frac{\partial\phi}{\partial r}\right)^2 + \left(\frac{\partial\phi}{\partial z}\right)^2$$

Taking the derivatives of ϕ with respect to r and z, and squaring yields

$$(\nabla\phi)^2 = \phi_o^2\left[\alpha^2 J_1^2(\alpha r)\cos^2(\beta z) + \beta^2 J_o(\alpha r)\sin^2(\beta z)\right]$$

Therefore, I_5 can be written as

$$I_5 = 2\pi\delta D\phi_o^2\left\{\alpha^2\int_o^R r J_1^2(\alpha r)\,dr\int_h^{h+\delta h}\cos^2(\beta z)\,dz + \beta^2\int_o^R r J_o^2(\alpha r)\,dr + \int_h^{h+\delta h}\sin^2(\beta z)\,dz\right\}$$

$$\alpha^2\int_o^R r J_1^2(\alpha r)\,dr = \frac{(\alpha R)^2}{2} J_1^2(\alpha R)$$

$$\int_h^{h+\delta h}\cos^2(\beta z)\,dz = \frac{HQ}{2\pi}$$

$$\int_o^R r J_o^2(\alpha r)\,dr = \frac{R^2}{2} J_1^2(\alpha R)$$

$$\int_h^{h+\delta h}\sin^2(\beta z)\,dz = \frac{HY}{2\pi}$$

where $$Y = \frac{\pi\delta h}{H} + \frac{1}{2}\sin\frac{2\pi h}{H} - \frac{1}{2}\sin\frac{2\pi(h+\delta h)}{H}$$

Therefore,

$$I_5 = 2\pi \delta D \phi_o^2 \left\{ \frac{(\alpha R)^2}{2} J_1^2(\alpha R) \frac{HQ}{2\pi} + \left(\frac{\pi}{H}\right)^2 \frac{R^2}{2} J_1^2(\alpha R) \frac{HY}{2\pi} \right\}$$

or

$$I_5 = \frac{1}{2} \delta D \phi_o^2 R^2 J_1^2(\alpha R) H \left[\alpha^2 Q + \left(\frac{\pi}{H}\right)^2 Y\right]$$

The evaluation of the integral in the denomenator of the reactivity equation is

$$I_6 = \int_v \nu \epsilon P C \Sigma_f \phi^2 dV = 2\pi \nu \epsilon P C \Sigma_f \phi_o^2 \int_0^R r J_0^2(\alpha R) dr \int_0^H \cos^2(\beta Z) dZ$$

$$\int_0^R r J_0^2(\alpha r) dr = \frac{R^2}{2} J_1^2(\alpha R)$$

$$\int_0^H \cos^2(\beta Z) dZ = \frac{H}{2}$$

Therefore,

$$I_6 = \frac{\pi}{2} \nu \epsilon P C \Sigma_f \phi_o^2 R^2 J_1^2(\alpha R) H$$

APPENDIX D

NUMERICAL EVALUATION OF REACTOR CORE PARAMETERS

A. RESONANCE ESCAPE PROBABILITY

A general expression given by Glasstone [4] for P, the resonance escape probability, for a thermal-neutron reactor is

$$P(E) = \exp\left[\frac{1}{\xi}\int_{E}^{E_0}\frac{\Sigma_a}{\Sigma_a+\Sigma_s}\,\frac{dE}{E}\right] \tag{36}$$

For a homogeneous reactor system, in which Σ_a is assumed to refer to the fuel only, and Σ_s is taken to be independent of neutron energy in the resonance region, (36) becomes

$$P(E) = \exp\left[-\frac{N^U}{\xi\Sigma_s}\int_{E}^{E_0}\left(\frac{\Sigma_s}{\Sigma_a+\Sigma_s}\sigma_a^U\right)\frac{dE}{E}\right] \tag{37}$$

The integral in (37) is called the effective resonance integral and can be represented by

$$I = \int_{E}^{E_0}\left(\frac{\Sigma_s}{\Sigma_a+\Sigma_s}\sigma_a^U\right)\frac{dE}{E}$$

Rewriting equation (37)

$$P = \exp\left[-\frac{N^U}{\Sigma_s}\,\frac{I}{\xi}\right] \tag{38}$$

For the AGN-201 reactor,

$$\Sigma_s = \Sigma_s^P + \Sigma_s^U = N^P\sigma_s^P + N^U\sigma_s^U$$

Dividing by N^U,

$$\frac{\Sigma_s}{N^U} = \frac{N^P}{N^U}\sigma_s^P + \sigma_s^U$$

However, since σ_s^U is small and can be neglected with respect to the first term,

$$\frac{\Sigma_s}{N^U} \approx \frac{N^P}{N^U}\sigma_s^P = \frac{\Sigma_s^P}{N^U} \tag{39}$$

Because $$\Sigma_s^P = \frac{\rho^P N_a}{A^P}\bar{\sigma}_a^P$$

and $$N^U = \frac{\rho^U N_a}{A^U}$$

equation (39) can be expressed as

$$\frac{\Sigma_s}{N^U} = \frac{\rho^P}{\rho^U}\left(\frac{\bar{\sigma}_s}{A}\right)^P A^U \tag{40}$$

From Appendix A,

$$\rho^P = 0.9383 \text{ gm-cm}^{-3}$$

and $$\rho^U = 0.2736 \text{ gm-cm}^{-3}$$

The evaluation of $\left(\frac{\bar{\sigma}_s}{A}\right)^P$ is as follows. The exact molecular weight of polyethylene, A^P, is unknown but can be written in terms of the $(CH_2)_n$ groups forming the molecule as

$$A^P = nA^{CH_2} = 14.0\ n \tag{41}$$

Similarly, σ_s can be written as

$$\sigma_s = n\sigma_s^C + 2n\sigma_s^H \tag{42}$$

Dividing (42) by (41) eliminates n and hence the exact molecular weight need not be known to find $\left(\frac{\sigma_s}{A}\right)^P$. This same method is employed to find the ratio of absorption cross-section to molecular weight in a subsequent paragraph. Correcting the cross-sections in (42) to an effective neutron temperature of 340° K through the relation

$$\bar{\sigma}_s = \frac{1}{1.128}\sqrt{\frac{T_m}{T_{eff}}}\,\sigma_s$$

yields

$$\bar{\sigma}_s^C = 3.85 \text{ b}$$

$$\sigma_s^H = 16.8 \text{ b}$$

Therefore, $$\left(\frac{\bar{\sigma}_s}{A}\right) = \frac{\bar{\sigma}_s^C + 2\bar{\sigma}_H^C}{A^P} = 2.67 \text{ b}$$

Also,

$$A^U = 270$$

Substitution of these values into (40) yields

$$\frac{\Sigma_s}{N^U} = 2472 \tag{43}$$

An emperical relationship for the effective resonance integral in which U-238 is the resonance absorber is given by Lamarsh [5] as

$$I = 2.73 \left(\frac{\Sigma_s}{N^U}\right)^{0.486} \tag{44}$$

Substituting (43) into (44)

$$I = 121.7 \tag{45}$$

The average logarithmic energy decrement per collision, $\bar{\xi}$, for a mixture of compounds such as in the fuel core of the AGN-201 is given as

$$\bar{\xi} = \frac{(\Sigma_s \xi)^U + (\Sigma_s \xi)^P}{\Sigma_s^U + \Sigma_s^P} \tag{46}$$

$$\Sigma_s^U = \frac{\rho^U N_a}{A^U} \bar{\sigma}_s^U = 0.0084 \text{ cm}^{-1}$$

where $\bar{\sigma}_s^U = 13.7$ b.

$$\xi^U = 0.036$$

$$\Sigma_s^P = \rho^P N_a \left(\frac{\bar{\sigma}_s}{A}\right)^P = 1.508 \text{ cm}^{-1}$$

The expression for ξ^P is

$$\xi^P = \frac{(\xi\bar{\sigma}_s)^C + 2(\xi\bar{\sigma}_s)^H}{\bar{\sigma}_s^C + 2\bar{\sigma}_s^H}$$

Using

$$\xi^C = 0.158$$

$$\xi^H = 1.00$$

the value for ξ^P is

$$\xi^P = 0.914$$

Substituting these values into (44) yields

$$\bar{\xi} = 0.9090 \qquad (47)$$

Inserting (43), (45), and (47) into (38), the value of the resonance escape probability term is found to be

$$P = 0.9473$$

B. FISSION CROSS-SECTION

The equation used to calculate the fission cross-section, Σ_f, is

$$\Sigma_f = \frac{\rho^{235} N_a}{A^{235}} \bar{\sigma}_f$$

From Appendix A,

$$\rho^{235} = 0.05451 \text{ gm-cm}^{-3}$$

The microscopic fission cross-section, corrected to a neutron temperature of 340° K is

$$\bar{\sigma}_f = 466 \text{ b.}$$

Since

$$N_a = 0.602$$

and

$$A^{235} = 235$$

the fission cross-section for the overall core is

$$\Sigma_f = 0.06507 \text{ cm}^{-1}$$

C. INFINITE MULTIPLICATION FACTOR

The expression for the infinite multiplacation factor, K_∞, is known as the four factor formula and is given as

$$K_\infty = \eta \epsilon p f$$

The value of η can be found from the equation

$$\eta = \nu^{235} \frac{\Sigma_f}{\Sigma_a^F}$$

Since in this expression, the "fuel" refers to the mixture of U-235 and U-238, Σ_a^F is given by

$$\Sigma_a^F = \Sigma_a^{235} + \Sigma_a^{238}$$

Therefore, $$\eta = \nu^{235} \frac{\Sigma_f}{\Sigma_a^{235} + \Sigma_a^{238}}$$

Since $$\nu^{235} = 2.47$$

$$\Sigma_a^{235} = \frac{\rho^{235} N_a}{A^{235}} \bar{\sigma}_a^{235} = 0.07764 \text{ cm}^{-1}$$

where $$\bar{\sigma}_a^{235} = 556 \text{ b}$$

and $$\Sigma_a^{238} = \frac{\rho^{238} N_a}{A^{238}} \bar{\sigma}_a^{238} = 0.001236 \text{ cm}^{-1}$$

where $$\bar{\sigma}_a^{238} = 2.23 \text{ b}$$

$$\eta = 2.038$$

It is assumed that the fast fission factor, ϵ, is equal to unity, based on the assumption that fast fission of U-235 does not contribute to the multiplication. From a previous paragraph

$$P = 0.9473$$

Finally, the equation for the thermal utilization term, f, is

$$f = \frac{\Sigma_a^f}{\Sigma_a} = \frac{\Sigma_a^u}{\Sigma_a^u + \Sigma_a^p}$$

$$\Sigma_a^u = \Sigma_a^{235} + \Sigma_a^{238} = 0.07888 \text{ cm}^{-1}$$

$$\Sigma_a^f = \rho^p N_a \left(\frac{\bar{\sigma}_a}{A}\right)^p = 0.02209 \text{ cm}^{-1}$$

where $$\left(\frac{\bar{\sigma}_a}{A}\right)^p = 0.0391 \text{ b.}$$

The resulting value for f is then

$$f = 0.7810.$$

Inserting these values into the four factor formula for K_∞, the infinite multiplication is found to be

$$K_\infty = 1.508$$

D. ABSORPTION CROSS-SECTION

The total absorption cross-section for the fuel core is found by summing the cross-section of all the substances present in the core. Therefore,

$$\Sigma_a = \Sigma_a^{235} + \Sigma_a^{238} + \Sigma_a^{P} = \Sigma_a^{U} + \Sigma_a^{P}$$

$$\Sigma_a = 0.1010 \text{ cm}^{-1}$$

E. DIFFUSION COEFFICIENT

From an earlier discussion concerning extrapolation distance, the assumption was made that without introducing significant error to the calculations, D could be defined as

$$D = \frac{1}{3\Sigma_s}$$

where $$\Sigma_s = \Sigma_s^{U} + \Sigma_s^{P} = \frac{\rho^{U} N_a}{A^{U}} \bar{\sigma}_s^{U} + \rho^{P} N_a \left(\frac{\bar{\sigma}_s}{A}\right)^{P}$$

since $$\bar{\sigma}_s^{U} = 13.7 \text{ b},$$

$$\Sigma_s = 1.516 \text{ cm}^{-1}$$

and the diffusion coefficient equals

$$D = 0.2199 \text{ cm}$$

F. DIFFUSION LENGTH

From diffusion theory, the quantity referred to as the diffusion length is defined as

$$L^2 = \frac{D}{\Sigma_a}$$

Since Σ_a and D have both been previously calculated

$$L^2 = 2.177 \text{ cm}^2$$

G. FERMI AGE TERM

The Fermi Age term must be evaluated last because it is solved in terms of all the other parameters in the criticality equation. Recall that the quantity C has been defined as

$$C = \frac{\Sigma_0}{B} \tan^{-1}\left(\frac{B}{\Sigma_0}\right) \tag{48}$$

For a cylindrical reactor core, B is given by the relation

$$B^2 = \alpha^2 + \beta^2$$

where $\alpha = \frac{2.405}{R} = 0.1894 \text{ cm}^{-1}$

and $\beta = \frac{\pi}{H} = 0.1302 \text{ cm}^{-1}$

Therefore, $B = 0.2298$

By definition,

$$\Sigma_0 = \frac{1}{(3\tau_0)^{1/2}} \tag{49}$$

The problem involved when dealing with an hydrogenous moderator, such as polyethylene, is that τ_0 cannot be calculated exactly from the Fermi Age equation and approximate methods must be resorted to. If one were to write the critical reactor equation

$$\frac{K_\infty C}{1 + L^2 B^2} = 1$$

to include the built-in excess reactivity of 0.000354 of the core, the resulting "supercritical" equation would be

$$\frac{K_\infty C}{1 + L^2 B^2} = 1.0004$$

Since the only unknown in this equation is C, the solution is

$$C = 0.1704$$

From (48),

$$\frac{\Sigma_0}{B} \tan^{-1}\left(\frac{B}{\Sigma_0}\right) = 0.1704.$$

Since B is known, this is an implicit function in terms of Σ_0. An iterative solution yields

$$\Sigma_o = 0.1985 \text{ cm}^{-1}$$

Therefore from (47), $\tau_o = 8.459 \text{ cm}^2$.

H. SUMMARY

The results of the preceding paragraphs may be tabulated as

$$P = 0.9473$$

$$\Sigma_f = 0.06507 \text{ cm}^{-1}$$

$$K_\infty = 1.508$$

$$\Sigma_a = 0.1010 \text{ cm}^{-1}$$

$$D = 0.2199 \text{ cm}$$

$$L^2 = 2.177 \text{ cm}^2$$

$$C = 0.1704$$

$$\tau_o = 8.459 \text{ cm}^2$$

APPENDIX E

COMPUTER PROGRAM 1

COMPUTER NOTATION

A5 = σ_a^{235} at neutron temperature of disk

A8 = σ_a^{238} at neutron temperature of disk

A235 = atomic weight of U-235

A238 = atomic weight of U-238

AC5 - σ_a^{235} at neutron temperature of 340°K

AC8 = σ_a^{238} at neutron temperature of 340° K

AN = Avogadro's number

APA = $\left(\frac{\sigma_a}{A}\right)^p$

AUO2 = atomic weight of uranium dioxide

B = buckling

CCD = C for a fuel disk with change in moderator mass

CD = C for a fuel disk

DD = D for a fuel disk

DEAP = $\delta\Sigma_a^p$

DELC = δC

DELD = δD

DELEA = $\delta\Sigma_a$

DELEF = $\delta\Sigma_f$

DELH = disk thickness

DELP = $\delta\rho$

DELPLC = change in density of polyethylene within the fuel core

DELPLD = change in density of polyethylene within a fuel disk

DELSA5 = $\delta\sigma_a^{235}$

DELSA8 = $\delta\sigma_a^{238}$

DELSF = $\delta\sigma_f$

DELTA = Δ

DENP = density of polyethylene in a disk

DENPLD = density of polyethylene in a disk with change in moderator mass

DENUO2 = density of uranium dioxide in a disk

DENU5 = density of U-235 in a disk

DENU8 = density of U-238 in a disk

DDD = D for a fuel disk with change in moderator mass

EAP = Σ_a^p

EAU = Σ_a^u

EO = Σ_o

EPA = Σ_a^p

ERI = effective resonance integral

ESP = Σ_s^p

EST = Σ_s

ESUO2 = Σ_s^u

GA5 = non-1/v absorption factor for U-235

GA8 = non-1/v absorption factor for U-238

GF = non-1/v fission factor for U-235

H = distance of nearest edge of fuel disk above or below the zero ordinate of the Z-axis of the cylindrical fuel core

HH = height of fuel core

HP = extrapolated height of fuel core

HR = equivalent height of reflected fuel core

PD = P for a disk

PM = mass of polyethylene in a fuel disk

PPD = P for a disk with change in moderator mass

RHOD = reactivity of fuel disk

RHOR1 = reactivity of fuel core for bare core geometry model

RHOR2 = reactivity of fuel core for extrapolated dimensions model

RHOR3 = reactivity of fuel core for reflected core model

SA50 = σ_a^{235} at neutron temperature of 293° K

SA80 = σ_a^{238} at neutron temperature of 293° K

SABAR5 = $\bar{\sigma}_a^{235}$

SABAR8 = $\bar{\sigma}_a^{238}$

SF = σ_f

SFBAR = $\bar{\sigma}_f$

SFO = σ_f at neutron temperature of 293° K

SIBAR = $\bar{\xi}$

SIP = ξ^P

SIU02 = ξ^U

SN = $\frac{\Sigma_s}{N^U}$

SPA = $\left(\frac{\sigma_3}{A}\right)^P$

SUO2 = σ_3^U

TEFF = effective neutron temperature

TM = moderator temperature

TO = T_0

VD = volume of disk

```
C       MAIN PROGRAM
        DIMENSION DENP(10),DENU02(10),DENU8(10),PD(10),CD(10),EPA(10),DD(1
       10),DELPLD(10),PM(10),VD(10),H(10),DELH(10),DENPLD(10),DENU5(10),SE
       1(10),A5(10),A8(10)
        READ(5,10)  (DENP(I),I=1,10),(DENU5(I),I=1,10),(DENU8(I),I=1,10),(
       1PM(I),I=1,10),(VD(I),I=1,10),(H(I),I=1,10),(DELH(I),I=1,10)
   10 FORMAT(10F8.0)
        DATA AU02,AN,SU02,SPA,SIU02,SIP,B,A235,A238,APA,TM,SED,GE,SA50,GA5
       1,SA80,GA8,AC5,AC8/270.,.602,13.7,2.67,.034,.914,.2298,235.,238.,.0
       1391,293.,532.,.9759,434.,.978,2.71,1.0017,571.,2.23/
        DO 20 I=1,10
        DENU02(I)=DENU5(I)+DENU8(I)
        SN=DENP(I)/DENU02(I)*SPA*AU02
        ZZ=0.486*ALOG(SN)
        ERI=2.73*EXP(ZZ)
        ESU02=DENU02(I)*AN*SU02/AU02
        ESP=DENP(I)*AN*SPA
        EST=ESU02+ESP
        SIBAR=(ESU02*SIU02+ESP*SIP)/EST
        PD(I)=1./EXP(ERI/(SN*SIBAR))
        TO=17.76/(SIBAR*EST**2)
        EO=1./SQRT(3.*TO)
        CD(I)=EO*ATAN(B/EO)/B
        EAU=DENU5(I)*AN*AC5/A235+DENU8(I)*AN*AC8/A238
        EAP=DENP(I)*AN*APA
        DELTA=4.*(EAU+EAP)/(SIBAR*EST)
        TEFF=TM*(1.+0.5459*DELTA)
        SE(I)=SQRT(TM/TEFF)*SED*GE/1.128
        A5(I)=SQRT(TM/TEFF)*SA50*GA5/1.128
        A8(I)=SQRT(TM/TEFF)*SA80*GA8/1.128
        EPA(I)=EAP
        DD(I)=1./(3.*EST)
   20 CONTINUE
        DELPLC=10.0
   30 DELPLD(1)=0.005847*DELPLC
        DELPLD(2)=0.02464*DELPLC
        DELPLD(3)=0.04828*DELPLC
        DELPLD(4)=0.08256*DELPLC
        DELPLD(5)=0.1243*DELPLC
        DELPLD(6)=0.1576*DELPLC
        DELPLD(7)=0.1893*DELPLC
        DELPLD(8)=0.1860*DELPLC
        DELPLD(9)=0.1332*DELPLC
        DELPLD(10)=0.04829*DELPLC
        DO 40 I=1,10
        DENPLD(I)=(PM(I)-DELPLD(I))/VD(I)
   40 CONTINUE
        DO 70 I=1,10
```

```
      SN=DENPLD(I)/DENUO2(I)*SPA*AUO2
      ZZ=0.486*ALOG(SN)
      ERI=2.73*EXP(ZZ)
      ESUO2=DENUO2(I)*AN*SUO2/AUO2
      ESP=DENPLD(I)*AN*SPA
      EST=ESUO2+ESP
      SIBAR=(ESUO2*SIUO2+ESP*SIP)/EST
      PPD=1./EXP(ERI/(SN*SIBAR))
      DELP=PPD-PD(I)
      TO=17.75/(SIBAR*EST**2)
      EO=1./SQRT(3.*TO)
      CCD=EO*ATAN(B/EO)/B
      DELC=CCD-CD(I)
      EAU=DENU5(I)*AN*AC5/A235+DENU8(I)*AN*AC8/A238
      EAP=DENPLD(I)*AN*APA
      DELTA=4.*(EAU+EAP)/(SIBAR*EST)
      TEFF=TM*(1.+0.5450*DELTA)
      SFBAR=SQRT(TM/TEFF)*SFO*GF/1.128
      DELSF=SFBAR-SF(I)
      DELEF=DENU5(I)*AN*DELSF/A235
      SABAR5=SQRT(TM/TEFF)*SA50*GA5/1.128
      DELSA5=SABAR5-A5(I)
      SABAR8=SQRT(TM/TEFF)*SA80*GA8/1.128
      DELSA8=SABAR8-A8(I)
      DEAP=DENPLD(I)*AN*APA-EPA(I)
      DELEA=DEAP+DENU5(I)*AN*DELSA5/A235+DENU8(I)*AN*DELSA8/A238
      DDD=1./(3.*EST)
      DELD=DDD-DD(I)
      N=I
      IF(N.GT.1) GO TO 60
      WRITE(6,50)
   50 FORMAT(11X,  ' BARE CORE SOLUTION',15X,  ' EXTRAPOLATED DIMENSIONS
     1 SOLUTION',25X,  ' REFLECTED CORE SOLUTION',//)
   60 CALL BCORE (DELP,DELC,DELEF,DELEA,DELD,N,DELPLC,H,DELH,RHOR1)
      CALL EXTDIM (DELP,DELC,DELEF,DELEA,DELD,N,DELPLC,H,DELH,RHOR2)
      CALL REFLCT (DELP,DELC,DELEF,DELEA,DELD,N,DELPLC,H,DELH,RHOR3)
   70 CONTINUE
      WRITE(6,80) DELPLC,RHOR1
   80 FORMAT(  ' REACTIVITY OF CORE (BARE CORE SOLUTION) FOR CHANGE IN D
     1ENSITY OF',1X,F4.0,  ' GRAMS =',F10.6,//)
      WRITE(6,90) DELPLC,RHOR2
   90 FORMAT(  ' REACTIVITY OF CORE (EXTRAPOLATED DIMENSIONS SOLUTION) F
     1OR CHANGE IN DENSITY OF',1X,F4.0,  ' GRAMS =',F10.6,//)
      WRITE(6,100) DELPLC,RHOR3
  100 FORMAT(  ' REACTIVITY OF CORE (REFLECTED CORE SOLUTION) FOR CHANGE
     1 IN DENSITY OF',1X,F4.0,  ' GRAMS =',F10.6)
      WRITE(6,110)
  110 FORMAT(1H1)
```

```
      DELPLC=DELPLC+10.0
      IF(DELPLC.GT.50.) GO TO 120
      GO TO 30
  120 STOP
      END



      SUBROUTINE BCORE (DELP,DELC,DELEF,DELFA,DELD,N,DELPLC,H,DELH,RHOR1
     1)
      DIMENSION H(10),DELH(10),RHOD(10)
      HH=9.5
      Q=(3.142*DELH(N)/HH)+0.5*SIN(6.284*(H(N)+DELH(N))/HH)-0.5*SIN(6.28
     14*H(N)/HH)
      Y=(3.142*DELH(N)/HH)-0.5*SIN(6.284*(H(N)+DELH(N))/HH)+0.5*SIN(6.28
     14*H(N)/HH)
      RHOD(N)=Q/3.142*(1.056*DELP+1.349*DELC+15.37*DELEF-8.844*DELFA-(0.
     13172+0.1499*Y/Q)*DELD)
      WRITE(6,10) N,RHOD(N)
   10 FORMAT(   ' DISK'1X,I2,4X,   ' REACTIVITY =',F10.6)
      IF(N.EQ.10) GO TO 20
      RETURN
   20 RHOR1=0.0
      DO 30 J=1,10
      RHOR1=RHOR1+RHOD(J)
   30 CONTINUE
      RETURN
      END



      SUBROUTINE EXTDIM (DELP,DELC,DELEF,DELFA,DELD,N,DELPLC,H,DELH,RHOR
     12)
      DIMENSION DELH(10),H(10),RHOD(10)
      HH=9.5
      HP=9.37
      A=3.142*HH/(2.*HP)+0.25*SIN(6.284*HH/HP)
      T=3.142*DELH(N)/HP+0.5*SIN(6.284*(H(N)+DELH(N))/HP)-0.5*SIN(6.284*
     1H(N)/HP)
      W=3.142*DELH(N)/HP-0.5*SIN(6.284*(H(N)+DELH(N))/HP)+0.5*SIN(6.284*
     14(N)/HP)
      RHOD(N)=(T/(2.*A))*(1.056*DELP+1.349*DELC+15.37*DELEF-8.844*DELFA-
     1(0.5265+0.1442*W/T)*DELD)
      WRITE(6,10) N,RHOD(N)
   10 FORMAT(45X,   ' DISK',1X,I2,4X,   ' REACTIVITY =',F10.6)
      IF(N.EQ.10) GO TO 20
```

```
      RETURN
   20 RHOR2=0.0
      DO 30 J=1,10
      RHOR2=RHOR2+RHOD(J)
   30 CONTINUE
      RETURN
      END



      SUBROUTINE REFLCT (DELP,DELC,DELEF,DELFA,DELD,N,DELPLC,H,DELH,RHOR
     13)
      DIMENSION DELH(10),H(10),RHOD(10)
      HH=9.5
      HR=12.25
      F= 3.142*DELH(N)/HR+0.5*SIN(6.284*(H(N)+DELH(N))/HR)-0.5*SIN(6.284
     1*H(N)/HR)
      S= 3.142*DELH(N)/HR-0.5*SIN(6.284*(H(N)+DELH(N))/HR)+0.5*SIN(6.284
     1*H(N)/HR)
      G=3.142*HH/(2.*HR)+0.25*SIN(6.284*HH/HR)
      RHOD(N)=(F/(2.*G))*(1.056*DELP+1.349*DELC+15.37*DELEF-2.644*DELFA-
     1(0.2297+0.09020*S/F)*DELD)
      WRITE(6,10) N,RHOD(N)
   10 FORMAT(85X,' DISK',1X,I2,4X,' REACTIVITY =',F10.6)
      IF(N.EQ.10) GO TO 20
      RETURN
   20 RHOR3=0.0
      DO 30 J=1,10
      RHOR3=RHOR3+RHOD(J)
   30 CONTINUE
      RETURN
      END
```

APPENDIX F

COMPUTER PROGRAM 2

A. COMPUTER NOTATION

The majority of the symbols appearing in this computer program are the same as in Computer Program 1 and have the same meaning. Therefore, reference should be made to Appendix E with the exception of the following terms.

DENPND = density of polyethylene in new fuel disk

DNU5ND = density of U-235 in new fuel disk

DNU8ND = density of U-238 in new fuel disk

DUO2ND = density of uranium dioxide in new fuel disk

RHOMAX = value of excess reactivity of the core after interchanging old and new disk

```
C       MAIN PROGRAM
        DIMENSION DENP(3),DENUO2(3),DENU8(3),DENU5(3),DELPLD(3),PD(3),CD(3
       1),EPA(3),DD(3),PM(3),H(3),SF(3),A5(3),A8(3),DELP(3),DELC(3),DELEF(
       13),DELEA(3),DELD(3)
        READ(5,10) (DENU5(I),I=1,3),(DENU8(I),I=1,3),(PM(I),I=1,3),(H(I),I
       1=1,3)
   10 FORMAT(10F8.0)
        DATA AUO2,AN,SUO2,SPA,SIUO2,SIP,B,A235,A238,APA,TM,SFO,GF,SA5O,GA5
       1,SA8O,GA8,AC5,AC8,VD,DELH,DENPND,DNU5ND,DNU8ND/270.,.602,13.7,2.67
       1,.036,.914,.2298,235.,238.,.0391,293.,582.,.9759,694.,.973,2.71,1.
       10017,571.,2.23,965.4,.75,1.268,0.06003,.2287/
        DELPLC=17.9
   20 DELPLD(1)=0.08256*DELPLC
        DELPLD(2)=0.1243*DELPLC
        DELPLD(3)=0.1576*DELPLC
        DO 30 I=1,3
        DENP(I)=(PM(I)-DELPLD(I))/VD
   30 CONTINUE
        DO 40 I=1,3
        DENUO2(I)=DENU5(I)+DENU8(I)
        SN=DENP(I)/DENUO2(I)*SPA*AUO2
        ZZ=0.486*ALOG(SN)
        ERI=2.73*EXP(ZZ)
        ESUO2=DENUO2(I)*AN*SUO2/AUO2
        ESP=DENP(I)*AN*SPA
        EST=ESUO2+ESP
        SIBAR=(ESUO2*SIUO2+ESP*SIP)/EST
        PD(I)=1./EXP(ERI/(SN*SIBAR))
        TO=17.75/(SIBAR*EST**2)
        EO=1./SQRT(3.*TO)
        CD(I)=EO*ATAN(B/EO)/B
        EAU=DENU5(I)*AN*AC5/A235+DENU8(I)*AN*AC8/A238
        EAP=DENP(I)*AN*APA
        DELTA=4.*(EAU+EAP)/(SIBAR*EST)
        TEFF=TM*(1.+0.5459*DELTA)
        SF(I)=SQRT(TM/TEFF)*SFO*GF/1.128
        A5(I)=SQRT(TM/TEFF)*SA5O*GA5/1.128
        A8(I)=SQRT(TM/TEFF)*SA8O*GA8/1.128
        EPA(I)=EAP
        DD(I)=1./(3.*EST)
   40 CONTINUE
        DO 50 I=1,3
        DUO2ND=DNU5ND+DNU8ND
        SN=DENPND/DUO2ND*SPA*AUO2
        ZZ=0.486*ALOG(SN)
        ERI=2.73*EXP(ZZ)
        ESUO2=DUO2ND*AN*SUO2/AUO2
        ESP=DENPND*AN*SPA
```

```
      EST=ESUO2+ESP
      SIBAR=(ESUO2*SIUO2+ESP*SIP)/EST
      PPD=1./EXP(ERI/(SN*SIBAR))
      DELP(I)=PPD-PD(I)
      TO=17.75/(SIBAR*EST**2)
      EO=1./SQRT(3.*TO)
      CCD=EC*ATAN(B/EO)/B
      DELC(I)=CCD-CD(I)
      EAU=DNU5ND*AN*AC5/A235+DNU8ND*AN*AC8/A238
      EAP=DENPND*AN*APA
      DELTA=4.*(EAU+EAP)/(SIBAR*EST)
      TEFF=TM*(1.+0.5459*DELTA)
      SFBAR=SQRT(TM/TEFF)*SFO*GF/1.128
      DELSF=SFBAR-SF(I)
      DLDNU5=DNU5ND-DENU5(I)
      DELEF(I)=DLDNU5*AN*DELSF/A235
      SABAR5=SQRT(TM/TEFF)*SA5O*GA5/1.128
      DELSA5=SABAR5-A5(I)
      SABAR8=SQRT(TM/TEFF)*SA8O*GA8/1.128
      DELSA8=SABAR8-A8(I)
      DLDNU8=DNU8ND-DENU8(I)
      DEAP=DENPND*AN*APA-EPA(I)
      DELEA(I)=DEAP+DLDNU5*AN*DELSA5/A235+DLDNU8*AN*DELSA8/A238
      DDD=1./(3.*EST)
      DELD(I)=DDD-DD(I)
   50 CONTINUE
      IF(DELPLC.EQ.17.9) GO TO 60
      IF(DELPLC.EQ.15.1) GO TO 90
      IF(DELPLC.EQ.13.8) GO TO 120
   60 DO 80 I=1,3
      N=I+3
      WRITE(6,70) N
   70 FORMAT(   ' REACTIVITY CHANGE RESULTING FROM INTERCHANGING NEW DISK
     1 WITH DISK',1X,I2,//)
      N=N-3
      CALL BCORE (DELP,DELC,DELEF,DELEA,DELD,N,H,DELH)
   80 CONTINUE
      DELPLC=15.1
      GO TO 20
   90 DO 110 I=1,3
      N=I+3
      WRITE(6,100) N
  100 FORMAT(   ' REACTIVITY CHANGE RESULTING FROM INTERCHANGING NEW DISK
     1 WITH DISK',1X,I2,//)
      N=N-3
      CALL EXTDIM (DELP,DELC,DELEF,DELEA,DELD,N,H,DELH)
  110 CONTINUE
      DELPLC=13.8
```

```
      GO TO 20
  120 DO 140 I=1,3
      N=I+3
      WRITE(6,130) N
  130 FORMAT(  ' REACTIVITY CHANGE RESULTING FROM INTERCHANGING NEW DISK
     1 WITH DISK',1X,I2,//)
      N=N-3
      CALL REFLCT (DELP,DELC,DELEF,DELEA,DELD,N,H,DELH)
  140 CONTINUE
      STOP
      END



      SUBROUTINE BCORE (DELP,DELC,DELEF,DELEA,DELD,N,H,DELH)
      DIMENSION H(3),DELP(3),DELC(3),DELEF(3),DELEA(3),DELD(3)
      HH=9.5
      Q=(3.142*DELH/HH)+0.5*SIN(6.284*(H(N)+DELH)/HH)-0.5*SIN(6.284*H(N)
     1/HH)
      Y=(3.142*DELH/HH)-0.5*SIN(6.284*(H(N)+DELH)/HH)+0.5*SIN(6.284*H(N)
     1/HH)
      RHOD=Q/3.142*(1.056*DELP(N)+1.349*DELC(N)+15.37*DELEF(N)-8.844*DEL
     1EA(N)-(0.3172+0.1499*Y/Q)*DELD(N))
      WRITE(6,10) RHOD
   10 FORMAT(  ' BARE CORE SOLUTION-  REACTIVITY =',F10.6)
      RHOMAX=0.00281+RHOD
      WRITE(6,20) RHOMAX
   20 FORMAT(21X,  ' MAXIMUM EXCESS REACTIVITY =',F10.6,//)
      RETURN
      END



      SUBROUTINE EXTDIM(DELP,DELC,DELEF,DELEA,DELD,N,H,DELH)
      DIMENSION H(3),DELP(3),DELC(3),DELEF(3),DELEA(3),DELD(3)
      HH=9.5
      HP=9.87
      A=3.142*HH/(2.*HP)+0.25*SIN(6.284*HH/HP)
      T=3.142*DELH/HP+0.5*SIN(6.284*(H(N)+DELH)/HP)-0.5*SIN(6.284*H(N)/H
     1P)
      W=3.142*DELH/HP-0.5*SIN(6.284*(H(N)+DELH)/HP)+0.5*SIN(6.284*H(N)/H
     1P)
      RHOD=(T/(2.*A))*(1.056*DELP(N)+1.349*DELC(N)+15.37*DELEF(N)-8.844*
     1DELEA(N)-(0.5265+0.1442*W/T)*DELD(N))
      WRITE(6,10) RHOD
   10 FORMAT(  ' EXTRAPOLATED DIMENSIONS SOLUTION-  REACTIVITY =',F10.6,
```

```
     1//)
      RHOMAX=0.00281+RHOD
      WRITE(6,20) RHOMAX
   20 FORMAT(35X,  ' MAXIMUM EXCESS REACTIVITY =',F10.6,//)
      RETURN
      END



      SUBROUTINE REFLCT(DELP,DELC,DELEF,DELEA,DELD,N,H,DELH)
      DIMENSION H(3),DELP(3),DELC(3),DELEF(3),DELEA(3),DELD(3)
      HH=9.5
      HR=12.25
      F=3.142*DELH/HR+0.5*SIN(6.284*(H(N)+DELH)/HR)-0.5*SIN(6.284*H(N)/H
     1R)
      S=3.142*DELH/HR-0.5*SIN(6.284*(H(N)+DELH)/HR)+0.5*SIN(6.284*H(N)/H
     1R)
      G=3.142*HH/(2.*HR)+0.25*SIN(6.284*HH/HR)
      RHOD=(F/(2.*G))*(1.056*DELP(N)+1.349*DELC(N)+15.37*DELEF(N)-8.844*
     1DELEA(N)-(0.2297+0.09020*S/F)*DELD(N))
      WRITE(6,10) RHOD
   10 FORMAT(  ' REFLECTED CORE SOLUTION-  REACTIVITY =',F10.6,//)
      RHOMAX=0.00281+RHOD
      WRITE(6,20) RHOMAX
   20 FORMAT(26X,  ' MAXIMUM EXCESS REACTIVITY =',F10.6,//)
      RETURN
      END
```

APPENDIX G

RESULTS OF COMPUTER PROGRAM 1

TABLE 6

Change in Moderator Core Mass = 10. gm

Disk	Reactivity (Bare Core)	Reactivity (Extrapolated Dimensions)	Reactivity (Reflected Core)
1	-0.000000	-0.000000	-0.000000
2	-0.000001	-0.000001	-0.000003
3	-0.000002	-0.000002	-0.000004
4	-0.000006	-0.000007	-0.000012
5	-0.000028	-0.000034	-0.000045
6	-0.000050	-0.000059	-0.000068
7	-0.000112	-0.000129	-0.000137
8	-0.000116	-0.000134	-0.000140
9	-0.000079	-0.000091	-0.000097
10	-0.000016	-0.000019	-0.000025

Reactivity of core (bare core solution) = -0.000410

Reactivity of core (extrapolated dimensions solution) = -0.000476

Reactivity of core (reflected core solution) = -0.000530

TABLE 7

Change in Moderator Core Mass = 20. gm

Disk	Reactivity (Bare Core)	Reactivity (Extrapolated Dimensions)	Reactivity (Reflected Core)
1	-0.000000	-0.000000	-0.000001
2	-0.000002	-0.000003	-0.000006
3	-0.000004	-0.000005	-0.000010
4	-0.000015	-0.000018	-0.000029
5	-0.000051	-0.000061	-0.000081
6	-0.000093	-0.000110	-0.000127
7	-0.000229	-0.000265	-0.000280
8	-0.000236	-0.000272	-0.000284
9	-0.000154	-0.000178	-0.000191
10	-0.000032	-0.000037	-0.000048

Reactivity of core (bare core solution) = -0.000818

Reactivity of core (extrapolated dimensions solution) = -0.000949

Reactivity of core (reflected core solution) = -0.001058

TABLE 8

Change in Moderator Core Mass = 30. gm

Disk	Reactivity (Bare Core)	Reactivity (Extrapolated Dimensions)	Reactivity (Reflected Core)
1	-0.000000	-0.000000	-0.000001
2	-0.000004	-.0000004	-0.000009
3	-0.000007	-0.000008	-0.000016
4	-0.000024	-0.000028	-0.000047
5	-0.000075	-0.000088	-0.000118
6	-0.000137	-0.000161	-0.000187
7	-0.000347	-0.000401	-0.000424
8	-0.000356	-0.000411	-0.000428
9	-0.000230	-0.000266	-0.000285
10	-0.000047	-0.000055	-0.000071

Reactivity of core (bare core solution) = -0.001227

Reactivity of core (extrapolated dimensions solution) = -0.001423

Reactivity of core (reflected core solution) = -0.001587

TABLE 9

Change in Moderator Core Mass = 40. gm

Disk	Reactivity (Bare Core)	Reactivity (Extrapolated Dimensions)	Reactivity (Reflected Core)
1	-0.000000	-0.000000	-0.000002
2	-0.000005	-0.000005	-0.000012
3	-0.000010	-0.000011	-0.000022
4	-0.000033	-0.000039	-0.000065
5	-0.000098	-0.000116	-0.000155
6	-0.000181	-0.000213	-0.000246
7	-0.000465	-0.000537	-0.000568
8	-0.000476	-0.000550	-0.000574
9	-0.000306	-0.000354	-0.000379
10	-0.000062	-0.000073	-0.000095

Reactivity of core (bare core solution) = -0.001638

Reactivity of core (extrapolated dimensions solution) = -0.001899

Reactivity of core (reflected core solution) = -0.002119

TABLE 10

Change in Moderator Core Mass = 50. gm

Disk	Reactivity (Bare Core)	Reactivity (Extrapolated Dimensions)	Reactivity (Reflected Core)
1	-0.000001	-0.000001	-0.000002
2	-0.000006	-0.000007	-0.000016
3	-0.000013	-0.000014	-0.000029
4	-0.000042	-0.000050	-0.000084
5	-0.000121	-0.000143	-0.000192
6	-0.000225	-0.000265	-0.000306
7	-0.000583	-0.000674	-0.000713
8	-0.000597	-0.000690	-0.000719
9	-0.000382	-0.000442	-0.000472
10	-0.000077	-0.000090	-0.000118

Reactivity of core (bare core solution) = -0.002049

Reactivity of core (extrapolated dimensions solution) = -0.002376

Reactivity of core (reflected core solution) = -0.002651

APPENDIX H

RESULTS OF COMPUTER PROGRAM 2

TABLE 11

Reactivity Change Resulting From
Interchanging New Disk With Disk 4

Model	Reactivity	Maximum Excess Reactivity
Bare Core	0.001582	0.004392
Extrapolated Dimensions	0.002006	0.004816
Reflected Core	0.002957	0.005767

TABLE 12

Reactivity Change Resulting From
Interchanging New Disk With Disk 5

Model	Reactivity	Maximum Excess Reactivity
Bare Core	0.002490	0.005300
Extrapolated	0.003128	0.005938
Reflected Core	0.003833	0.006643

TABLE 13

Reactivity Change Resulting From
Interchanging New Disk With Disk 6

Model	Reactivity	Maximum Excess Reactivity
Bare Core	0.003671	0.006481
Extrapolated Dimensions	0.004541	0.007351
Reflected Core	0.004897	0.007707

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13. ABSTRACT

The effects of fuel burnup, fission product poisoning, and hydrogen moderator density variation on reactivity in the AGN-201 reactor are considered. A modified one-group perturbation theory is developed and applied to changes in parameters resulting from a change in hydrogen moderator density. An equation for the reactivity change is obtained for three models: a bare cylindrical core, a bare core using extrapolated dimensions, and a reflected core. These three equations are then used to predict values of the reactivity increase resulting from interchanging a new 3/4 inch thick fuel disk with comparable fuel disks presently in the core.

The results obtained by a digital computer solution of the reactivity equations reveal that the increase in reactivity varies from 0.4392 to 0.7707%, depending upon the core model and position of the old disk within the core. Because the license of the Naval Postgraduate School does not permit a value of excess reactivity above 0.40%, it is concluded that a simple interchange of disks in this manner would produce too large a value of excess reactivity.

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Perturbation						
Polyethylene						
Fermi Age						
Extrapolation						
Neutron Temperature						
Samarium						